Chapter 15

15-1 Given: Uncrowned, through-hardened 300 Brinell core and case, Grade 1, $N_C = 10^9$ rev of pinion at $R = 0.999$, $N_P = 20$ teeth, $N_G = 60$ teeth, $Q_v = 6$, $P_d = 6$ teeth/in, shaft angle = 90°, $n_p = 900$ rev/min, $J_P = 0.249$ and $J_G = 0.216$ (Fig. 15-7), $F = 1.25$ in, $S_F = S_H = 1$, $K_o = 1$.

Mesh $d_P = 20/6 = 3.333$ in, $d_G = 60/6 = 10.000$ in

Eq. (15-7): $v_t = \pi(3.333)(900/12) = 785.3$ ft/min

Eq. (15-6): $B = 0.25(12 - 6)^{2/3} = 0.8255$
$A = 50 + 56(1 - 0.8255) = 59.77$

Eq. (15-5): $$K_v = \left(\frac{59.77 + \sqrt{785.3}}{59.77}\right)^{0.8255} = 1.374$$

Eq. (15-8): $v_{t,\max} = [59.77 + (6 - 3)]^2 = 3940$ ft/min

Since 785.3 < 3904, $K_v = 1.374$ is valid. The size factor for bending is:

Eq. (15-10): $K_s = 0.4867 + 0.2132 / 6 = 0.5222$

For one gear straddle-mounted, the load-distribution factor is:

Eq. (15-11): $K_m = 1.10 + 0.0036\,(1.25)^2 = 1.106$

Eq. (15-15): $(K_L)_P = 1.6831(10^9)^{-0.0323} = 0.862$
$(K_L)_G = 1.6831(10^9 / 3)^{-0.0323} = 0.893$

Eq. (15-14): $(C_L)_P = 3.4822(10^9)^{-0.0602} = 1$
$(C_L)_G = 3.4822(10^9 / 3)^{-0.0602} = 1.069$

Eq. (15-19): $K_R = 0.50 - 0.25 \log(1 - 0.999) = 1.25$ (or Table 15-3)
$C_R = \sqrt{K_R} = \sqrt{1.25} = 1.118$

Bending

Fig. 15-13: $_{0.99}S_t = s_{at} = 44(300) + 2100 = 15\ 300$ psi

Eq. (15-4): $$(\sigma_{\text{all}})_P = s_{wt} = \frac{s_{at}K_L}{S_F K_T K_R} = \frac{15\ 300(0.862)}{1(1)(1.25)} = 10\ 551 \text{ psi}$$

Eq. (15-3):
$$W_P^t = \frac{(\sigma_{\text{all}})_P F K_x J_P}{P_d K_o K_v K_s K_m}$$
$$= \frac{10\ 551(1.25)(1)(0.249)}{6(1)(1.374)(0.5222)(1.106)} = 690\ \text{lbf}$$
$$H_1 = \frac{690(785.3)}{33\ 000} = 16.4\ \text{hp}$$

Eq. (15-4):
$$(\sigma_{\text{all}})_G = \frac{15\ 300(0.893)}{1(1)(1.25)} = 10\ 930\ \text{psi}$$
$$W_G^t = \frac{10\ 930(1.25)(1)(0.216)}{6(1)(1.374)(0.5222)(1.106)} = 620\ \text{lbf}$$
$$H_2 = \frac{620(785.3)}{33\ 000} = 14.8\ \text{hp} \quad \textit{Ans.}$$

The gear controls the bending rating.

15-2 Refer to Prob. 15-1 for the gearset specifications.

Wear

Fig. 15-12: $s_{ac} = 341(300) + 23\ 620 = 125\ 920$ psi

For the pinion, $C_H = 1$. From Prob. 15-1, $C_R = 1.118$. Thus, from Eq. (15-2):

$$(\sigma_{c,\text{all}})_P = \frac{s_{ac}(C_L)_P C_H}{S_H K_T C_R}$$
$$(\sigma_{c,\text{all}})_P = \frac{125\ 920(1)(1)}{1(1)(1.118)} = 112\ 630\ \text{psi}$$

For the gear, from Eq. (15-16),

$$B_1 = 0.008\ 98(300\ /\ 300) - 0.008\ 29 = 0.000\ 69$$
$$C_H = 1 + 0.000\ 69(3 - 1) = 1.001\ 38$$

From Prob. 15-1, $(C_L)_G = 1.0685$. Equation (15-2) thus gives

$$(\sigma_{c,\text{all}})_G = \frac{s_{ac}(C_L)_G C_H}{S_H K_T C_R}$$
$$(\sigma_{c,\text{all}})_G = \frac{125\ 920(1.0685)(1.001\ 38)}{1(1)(1.118)} = 120\ 511\ \text{psi}$$

For steel: $C_p = 2290\sqrt{\text{psi}}$

Eq. (15-9): $C_s = 0.125(1.25) + 0.4375 = 0.593\ 75$

Fig. 15-6: $I = 0.083$

Eq. (15-12): $C_{xc} = 2$

Eq. (15-1): $W_P^t = \left(\frac{(\sigma_{c,\text{all}})_P}{C_p}\right)^2 \frac{Fd_P I}{K_o K_v K_m C_s C_{xc}}$

$$= \left(\frac{112\ 630}{2290}\right)^2 \left[\frac{1.25(3.333)(0.083)}{1(1.374)(1.106)(0.5937)(2)}\right]$$

$$= 464 \text{ lbf}$$

$$H_3 = \frac{464(785.3)}{33\ 000} = 11.0 \text{ hp}$$

$$W_G^t = \left(\frac{120\ 511}{2290}\right)^2 \left[\frac{1.25(3.333)(0.083)}{1(1.374)(1.106)(0.593\ 75)(2)}\right]$$

$$= 531 \text{ lbf}$$

$$H_4 = \frac{531(785.3)}{33\ 000} = 12.6 \text{ hp}$$

The pinion controls wear: $H = 11.0$ hp *Ans.*

The power rating of the mesh, considering the power ratings found in Prob. 15-1, is

$H = \min(16.4, 14.8, 11.0, 12.6) = 11.0$ hp *Ans.*

15-3 AGMA 2003-B97 does not fully address cast iron gears. However, approximate comparisons can be useful. This problem is similar to Prob. 15-1, but not identical. We will organize the method. A follow-up could consist of completing Probs. 15-1 and 15-2 with identical pinions, and cast iron gears.

Given: Uncrowned, straight teeth, $P_d = 6$ teeth/in, $N_P = 30$ teeth, $N_G = 60$ teeth, ASTM 30 cast iron, material Grade 1, shaft angle 90°, $F = 1.25$, $n_P = 900$ rev/min, $\phi_n = 20°$, one gear straddle-mounted, $K_o = 1$, $J_P = 0.268$, $J_G = 0.228$, $S_F = 2$, $S_H = \sqrt{2}$.

Mesh $d_P = 30/6 = 5.000$ in, $d_G = 60/6 = 10.000$ in

$v_t = \pi(5)(900 / 12) = 1178$ ft/min

Set $N_L = 10^7$ cycles for the pinion. For $R = 0.99$,

Table 15-7: $s_{at} = 4500$ psi

Table 15-5: s_{ac} = 50 000 psi

Eq. (15-4): $$s_{wt} = \frac{s_{at}K_L}{S_F K_T K_R} = \frac{4500(1)}{2(1)(1)} = 2250 \text{ psi}$$

The velocity factor K_v represents stress augmentation due to mislocation of tooth profiles along the pitch surface and the resulting "falling" of teeth into engagement. Equation (5-67) shows that the induced bending moment in a cantilever (tooth) varies directly with $\sqrt{E}$ of the tooth material. If only the material varies (cast iron vs. steel) in the same geometry, I is the same. From the Lewis equation of Section 14-1,

$$\sigma = \frac{M}{I/c} = \frac{K_v W^t P}{FY}$$

We expect the ratio $\sigma_{\text{CI}}/\sigma_{\text{steel}}$ to be

$$\frac{\sigma_{CI}}{\sigma_{\text{steel}}} = \frac{(K_v)_{\text{CI}}}{(K_v)_{\text{steel}}} = \sqrt{\frac{E_{CI}}{E_{\text{steel}}}}$$

In the case of ASTM class 30, from Table A-24(*a*)

$$(E_{\text{CI}})_{\text{av}} = (13 + 16.2)/2 = 14.7 \text{ kpsi}$$

Then, $$(K_v)_{\text{CI}} = \sqrt{\frac{14.7}{30}}(K_v)_{\text{steel}} = 0.7(K_v)_{\text{steel}}$$

Our modeling is rough, but it convinces us that $(K_v)_{\text{CI}} < (K_v)_{\text{steel}}$, but we are not sure of the value of $(K_v)_{\text{CI}}$. We will use K_v for steel as a basis for a conservative rating.

Eq. (15-6): $$B = 0.25(12 - 6)^{2/3} = 0.8255$$
$$A = 50 + 56(1 - 0.8255) = 59.77$$

Eq. (15-5): $$K_v = \left(\frac{59.77 + \sqrt{1178}}{59.77}\right)^{0.8255} = 1.454$$

Pinion bending $(\sigma_{\text{all}})_P = s_{wt} = 2250$ psi

From Prob. 15-1, $K_x = 1$, $K_m = 1.106$, $K_s = 0.5222$

Eq. (15-3): $$W_P^t = \frac{(\sigma_{\text{all}})_P F K_x J_P}{P_d K_o K_v K_s K_m}$$
$$= \frac{2250(1.25)(1)(0.268)}{6(1)(1.454)(0.5222)(1.106)} = 149.6 \text{ lbf}$$

$$H_1 = \frac{149.6(1178)}{33\ 000} = 5.34 \text{ hp}$$

Gear bending

$$W_G^t = W_P^t \frac{J_G}{J_P} = 149.6\left(\frac{0.228}{0.268}\right) = 127.3 \text{ lbf}$$

$$H_2 = \frac{127.3(1178)}{33\ 000} = 4.54 \text{ hp}$$

The gear controls in bending fatigue. H = 4.54 hp *Ans.*

15-4 Continuing Prob. 15-3,

Table 15-5: s_{ac} = 50 000 psi

$$s_{wt} = \sigma_{c,\text{all}} = \frac{50\ 000}{\sqrt{2}} = 35\ 355 \text{ psi}$$

Eq. (15-1): $$W^t = \left(\frac{\sigma_{c,\text{all}}}{C_p}\right)^2 \frac{Fd_P I}{K_o K_v K_m C_s C_{xc}}$$

Fig. 15-6: I = 0.86

From Probs. 15-1 and 15-2: C_s = 0.593 75, K_s = 0.5222, K_m = 1.106, C_{xc} = 2

From Table 14-8: $C_p = 1960\sqrt{\text{psi}}$

Thus, $$W^t = \left(\frac{35\ 355}{1960}\right)^2 \left[\frac{1.25(5.000)(0.086)}{1(1.454)(1.106)(0.59375)(2)}\right] = 91.6 \text{ lbf}$$

$$H_3 = H_4 = \frac{91.6(1178)}{33\ 000} = 3.27 \text{ hp}$$

Rating

Based on results of Probs. 15-3 and 15-4,

H = min(5.34, 4.54, 3.27, 3.27) = 3.27 hp *Ans.*

The mesh is weakest in wear fatigue.

15-5 Uncrowned, through-hardened to 180 Brinell (core and case), Grade 1, 10^9 rev of pinion at R = 0.999, $N_P = z_1$ = 22 teeth, $N_G = z_2$ = 24 teeth, Q_v = 5, m_{et} = 4 mm, shaft angle 90°, n_1 = 1800 rev/min, S_F = 1, $S_H = \sqrt{S_F} = \sqrt{1}$, $J_P = Y_{J1}$ = 0.23, $J_G = Y_{J2}$ = 0.205, $F = b$ = 25 mm, $K_o = K_A = K_T = K_\theta$ = 1 and $C_p = 190\sqrt{\text{MPa}}$.

Mesh $d_P = d_{e1} = mz_1 = 4(22) = 88$ mm, $d_G = m_{et}\, z_2 = 4(24) = 96$ mm

Eq. (15-7): $v_{et} = 5.236(10^{-5})(88)(1800) = 8.29$ m/s

Eq. (15-6): $B = 0.25(12 - 5)^{2/3} = 0.9148$
$A = 50 + 56(1 - 0.9148) = 54.77$

Eq. (15-5): $$K_v = \left(\frac{54.77 + \sqrt{200(8.29)}}{54.77} \right)^{0.9148} = 1.663$$

Eq. (15-10): $K_s = Y_x = 0.4867 + 0.008\,339(4) = 0.520$

Eq. (15-11): with $K_{mb} = 1$ (both straddle-mounted),
$K_m = K_{H\beta} = 1 + 5.6(10^{-6})(25^2) = 1.0035$

From Fig. 15-8,
$$(C_L)_P = (Z_{NT})_P = 3.4822(10^9)^{-0.0602} = 1.00$$
$$(C_L)_G = (Z_{NT})_G = 3.4822[10^9(22\ /\ 24)]^{-0.0602} = 1.0054$$

Eq. (15-12): $C_{xc} = Z_{xc} = 2$ (uncrowned)

Eq. (15-19): $K_R = Y_Z = 0.50 - 0.25 \log (1 - 0.999) = 1.25$
$$C_R = Z_Z = \sqrt{Y_Z} = \sqrt{1.25} = 1.118$$

From Fig. 15-10, $C_H = Z_w = 1$

Eq. (15-9): $Z_x = 0.004\,92(25) + 0.4375 = 0.560$

Wear of Pinion

Fig. 15-12: $\sigma_{H\,\text{lim}} = 2.35H_B + 162.89$
$= 2.35(180) + 162.89 = 585.9$ MPa

Fig. 15-6: $I = Z_I = 0.066$

Eq. (15-2): $$(\sigma_H)_P = \frac{(\sigma_{H\ \text{lim}})_P (Z_{NT})_P Z_W}{S_H K_\theta Z_Z}$$
$$= \frac{585.9(1)(1)}{\sqrt{1}(1)(1.118)} = 524.1 \text{ MPa}$$

Eq. (15-1): $$W_P^t = \left(\frac{\sigma_H}{C_p} \right)^2 \frac{b d_{e1} Z_I}{1000 K_A K_v K_{H\beta} Z_x Z_{xc}}$$
The constant 1000 expresses W^t in kN.

$$W_P^t = \left(\frac{524.1}{190}\right)^2 \left[\frac{25(88)(0.066)}{1000(1)(1.663)(1.0035)(0.56)(2)}\right] = 0.591\text{ kN}$$

Eq. (13-36): $$H_3 = \frac{\pi d n_1 W^t}{60\,000} = \frac{\pi(88)(1800)(0.591)}{60\,000} = 4.90\text{ kW}$$

Wear of Gear

$$\sigma_{H\,\text{lim}} = 585.9\text{ MPa}$$

$$(\sigma_H)_G = \frac{585.9(1.0054)}{\sqrt{1}(1)(1.118)} = 526.9\text{ MPa}$$

$$W_G^t = W_P^t\frac{(\sigma_H)_G}{(\sigma_H)_P} = 0.591\left(\frac{526.9}{524.1}\right) = 0.594\text{ kN}$$

$$H_4 = \frac{\pi(88)(1800)(0.594)}{60\,000} = 4.93\text{ kW}$$

Thus in wear, the pinion controls the power rating; $H = 4.90$ kW *Ans.*

We will rate the gear set after solving Prob. 15-6.

15-6 Refer to Prob. 15-5 for terms not defined below.

Bending of Pinion

$$(K_L)_P = (Y_{NT})_P = 1.6831(10^9)^{-0.0323} = 0.862$$
$$(K_L)_G = (Y_{NT})_G = 1.6831[10^9(22/24)]^{-0.0323} = 0.864$$

Fig. 15-13: $\sigma_{F\,\text{lim}} = 0.30H_B + 14.48$
$= 0.30(180) + 14.48 = 68.5$ MPa

Eq. (15-13): $K_x = Y_\beta = 1$

From Prob. 15-5: $Y_Z = 1.25$, $v_{et} = 8.29$ m/s,
$K_A = 1$, $K_v = 1.663$, $K_\theta = 1$,
$Y_x = 0.52$, $K_{H\beta} = 1.0035$, $Y_{J1} = 0.23$

Eq. (5-4): $$(\sigma_F)_P = \frac{\sigma_{F\,\text{lim}}Y_{NT}}{S_F K_\theta Y_Z} = \frac{68.5(0.862)}{1(1)(1.25)} = 47.2\text{ MPa}$$

Eq. (5-3): $$W_P^t = \frac{(\sigma_F)_P b m_{et} Y_\beta Y_{J1}}{1000 K_A K_v Y_x K_{H\beta}}$$
$$= \frac{47.2(25)(4)(1)(0.23)}{1000(1)(1.663)(0.52)(1.0035)} = 1.25\text{ kN}$$

$$H_1 = \frac{\pi(88)(1800)(1.25)}{60\,000} = 10.37 \text{ kW}$$

Bending of Gear

$$\sigma_{F\,\text{lim}} = 68.5 \text{ MPa}$$

$$(\sigma_F)_G = \frac{68.5(0.864)}{1(1)(1.25)} = 47.3 \text{ MPa}$$

$$W_G^t = \frac{47.3(25)(4)(1)(0.205)}{1000(1)(1.663)(0.52)(1.0035)} = 1.12 \text{ kN}$$

$$H_2 = \frac{\pi(88)(1800)(1.12)}{60\,000} = 9.29 \text{ kW}$$

Rating of mesh is

$$H_{\text{rating}} = \min(10.37, 9.29, 4.90, 4.93) = 4.90 \text{ kW} \qquad \textit{Ans.}$$

with pinion wear controlling.

15-7

(a) $$(S_F)_P = \left(\frac{\sigma_{\text{all}}}{\sigma}\right)_P = (S_F)_G = \left(\frac{\sigma_{\text{all}}}{\sigma}\right)_G$$

$$\frac{(s_{at}K_L \;/\; K_T K_R)_P}{(W^t P_d K_o K_v K_s K_m \;/\; F K_x J)_P} = \frac{(s_{at}K_L \;/\; K_T K_R)_G}{(W^t P_d K_o K_v K_s K_m \;/\; F K_x J)_G}$$

All terms cancel except for s_{at} , K_L , and J,

$$(s_{at})_P (K_L)_P\, J_P = (s_{at})_G (K_L)_G\, J_G$$

From which

$$(s_{at})_G = \frac{(s_{at})_P (K_L)_P J_P}{(K_L)_G J_G} = (s_{at})_P \frac{J_P}{J_G} m_G^{\beta}$$

where $\beta = -0.0178$ or $\beta = -0.0323$ as appropriate. This equation is the same as Eq. (14-44). *Ans.*

(b) In bending

$$W^t = \left(\frac{\sigma_{\text{all}}}{S_F} \frac{F K_x J}{P_d K_o K_v K_s K_m}\right)_{11} = \left(\frac{s_{at}}{S_F} \frac{K_L}{K_T K_R} \frac{F K_x J}{P_d K_o K_v K_s K_m}\right)_{11} \qquad (1)$$

In wear

$$\left(\frac{s_{ac} C_L C_U}{S_H K_T C_R}\right)_{22} = C_p \left(\frac{W^t K_o K_v K_m C_s C_{xc}}{F d_P I}\right)_{22}^{1/2}$$

Squaring and solving for W^t gives

$$W^t = \left(\frac{s_{ac}^2 C_L^2 C_H^2}{S_H^2 K_T^2 C_R^2 C_P^2}\right)_{22} \left(\frac{F d_P I}{K_o K_v K_m C_s C_{xc}}\right)_{22} \qquad (2)$$

Equating the right-hand sides of Eqs. (1) and (2) and canceling terms, and recognizing that $C_R = \sqrt{K_R}$ and $P_d d_P = N_P$, we obtain

$$(s_{ac})_{22} = \frac{C_p}{(C_L)_{22}} \sqrt{\frac{S_H^2}{S_F} \frac{(s_{at})_{11}(K_L)_{11} K_x J_{11} K_T C_s C_{xc}}{C_H^2 N_P K_s I}}$$

For equal W^t in bending and wear

$$\frac{S_H^2}{S_F} = \frac{\left(\sqrt{S_F}\right)^2}{S_F} = 1$$

So we get

$$(s_{ac})_G = \frac{C_p}{(C_L)_G C_H} \sqrt{\frac{(s_{at})_P (K_L)_P J_P K_x K_T C_s C_{xc}}{N_P I K_s}} \qquad \textit{Ans.}$$

(c)

$$(S_H)_P = (S_H)_G = \left(\frac{\sigma_{c,\text{all}}}{\sigma_c}\right)_P = \left(\frac{\sigma_{c,\text{all}}}{\sigma_c}\right)_G$$

Substituting in the right-hand equality gives

$$\frac{[s_{ac} C_L / (C_R K_T)]_P}{\left[C_p \sqrt{W^t K_o K_v K_m C_s C_{xc} / (F d_P I)}\right]_P} = \frac{[s_{ac} C_L C_H / (C_R K_T)]_G}{\left[C_p \sqrt{W^t K_o K_v K_m C_s C_{xc} / (F d_P I)}\right]_G}$$

Denominators cancel, leaving

$$(s_{ac})_P (C_L)_P = (s_{ac})_G (C_L)_G C_H$$

Solving for $(s_{ac})_P$ gives,

$$(s_{ac})_P = (s_{ac})_G \frac{(C_L)_G}{(C_L)_P} C_H \qquad (1)$$

From Eq. (15-14), $(C_L)_P = 3.4822 N_L^{-0.0602}$ and $(C_L)_G = 3.4822 (N_L / m_G)^{-0.0602}$.
Thus,

$$(s_{ac})_P = (s_{ac})_G (1/m_G)^{-0.0602} C_H = (s_{ac})_G m_G^{0.0602} C_H \qquad \textit{Ans.}$$

This equation is the transpose of Eq. (14-45).

15-8

	Core	Case
Pinion	$(H_B)_{11}$	$(H_B)_{12}$
Gear	$(H_B)_{21}$	$(H_B)_{22}$

Given $(H_B)_{11}$ = 300 Brinell

Eq. (15-23): $(s_{at})_P = 44(300) + 2100 = 15\ 300$ psi

$$(s_{at})_G = (s_{at})_P \frac{J_P}{J_G} m_G^{-0.0323} = 15\ 300\left(\frac{0.249}{0.216}\right)\left(3^{-0.0323}\right) = 17\ 023 \text{ psi}$$

$$(H_B)_{21} = \frac{17\ 023 - 2100}{44} = 339 \text{ Brinell} \quad \textit{Ans.}$$

$$(s_{ac})_G = \frac{2290}{1.0685(1)}\sqrt{\frac{15\ 300(0.862)(0.249)(1)(0.593\ 25)(2)}{20(0.086)(0.5222)}}$$
$$= 141\,160 \text{ psi}$$

$$(H_B)_{22} = \frac{141\,160 - 23\ 600}{341} = 345 \text{ Brinell} \quad \textit{Ans.}$$

$$(s_{ac})_P = (s_{ac})_G m_G^{0.0602} C_H \doteq 141\,160(3^{0.0602})(1) = 150\ 811 \text{ psi}$$

$$(H_B)_{12} = \frac{150\ 811 - 23\ 600}{341} = 373 \text{ Brinell} \quad \textit{Ans.}$$

	Core	Case
Pinion	300	373
Gear	339	345

Ans.

15-9

Pinion core

$$(s_{at})_P = 44(300) + 2100 = 15\ 300 \text{ psi}$$

$$(\sigma_{\text{all}})_P = \frac{15\ 300(0.862)}{1(1)(1.25)} = 10\ 551 \text{ psi}$$

$$W^t = \frac{10\ 551(1.25)(0.249)}{6(1)(1.374)(0.5222)(1.106)} = 689.7 \text{ lbf}$$

Gear core

$$(s_{at})_G = 44(352) + 2100 = 17\ 588 \text{ psi}$$

$$(\sigma_{\text{all}})_G = \frac{17\ 588(0.893)}{1(1)(1.25)} = 12\ 565 \text{ psi}$$

$$W^t = \frac{12\ 565(1.25)(0.216)}{6(1)(1.374)(0.5222)(1.106)} = 712.5 \text{ lbf}$$

Pinion case

$$(s_{ac})_P = 341(372) + 23\,620 = 150\,472 \text{ psi}$$

$$(\sigma_{c,\text{all}})_P = \frac{150\,472(1)}{1(1)(1.118)} = 134\,590 \text{ psi}$$

$$W^t = \left(\frac{134\,590}{2290}\right)^2 \left[\frac{1.25(3.333)(0.086)}{1(1.374)(1.106)(0.593\,75)(2)}\right] = 685.8 \text{ lbf}$$

Gear case

$$(s_{ac})_G = 341(344) + 23\,620 = 140\,924 \text{ psi}$$

$$(\sigma_{c,\text{all}})_G = \frac{140\,924(1.0685)(1)}{1(1)(1.118)} = 134\,685 \text{ psi}$$

$$W^t = \left(\frac{134\,685}{2290}\right)^2 \frac{1.25(3.333)(0.086)}{1(1.374)(1.106)(0.593\,75)(2)} = 686.8 \text{ lbf}$$

The rating load would be

$$W^t_{\text{rated}} = \min(689.7,\ 712.5,\ 685.8,\ 686.8) = 685.8 \text{ lbf}$$

which is slightly less than intended.

Pinion core

$$(s_{at})_P = 15\,300 \text{ psi} \qquad \text{(as before)}$$

$$(\sigma_{\text{all}})_P = 10\,551 \text{ psi} \qquad \text{(as before)}$$

$$W^t = 689.7 \text{ lbf} \qquad \text{(as before)}$$

Gear core

$$(s_{at})_G = 44(339) + 2100 = 17\,016 \text{ psi}$$

$$(\sigma_{\text{all}})_G = \frac{17\,016(0.893)}{1(1)(1.25)} = 12\,156 \text{ psi}$$

$$W^t = \frac{12\,156(1.25)(0.216)}{6(1)(1.374)(0.5222)(1.106)} = 689.3 \text{ lbf}$$

Pinion case

$$(s_{ac})_P = 341(373) + 23\,620 = 150\,813 \text{ psi}$$

$$(\sigma_{c,\text{all}})_P = \frac{150\,813(1)}{1(1)(1.118)} = 134\,895 \text{ psi}$$

$$W^t = \left(\frac{134\,895}{2290}\right)^2 \left[\frac{1.25(3.333)(0.086)}{1(1.374)(1.106)(0.593\,75)(2)}\right] = 689.0 \text{ lbf}$$

Gear case

$$(s_{ac})_G = 341(345) + 23\,620 = 141\,265 \text{ psi}$$

$$(\sigma_{c,\text{all}})_G = \frac{141\,265(1.0685)(1)}{1(1)(1.118)} = 135\,010 \text{ psi}$$

$$W^t = \left(\frac{135\ 010}{2290}\right)^2 \left[\frac{1.25(3.333)(0.086)}{1(1.1374)(1.106)(0.593\ 75)(2)}\right] = 690.1 \text{ lbf}$$

The equations developed within Prob. 15-7 are effective.

15-10 The catalog rating is 5.2 hp at 1200 rev/min for a straight bevel gearset. Also given: $N_P = 20$ teeth, $N_G = 40$ teeth, $\phi_n = 20°$, $F = 0.71$ in, $J_P = 0.241$, $J_G = 0.201$, $P_d = 10$ teeth/in, through-hardened to 300 Brinell-General Industrial Service, and $Q_v = 5$ uncrowned.

Mesh

$$d_P = 20/10 = 2.000 \text{ in}, \quad d_G = 40/10 = 4.000 \text{ in}$$

$$v_t = \frac{\pi d_P n_P}{12} = \frac{\pi(2)(1200)}{12} = 628.3 \text{ ft/min}$$

$$K_o = 1, \quad S_F = 1, \quad S_H = 1$$

Eq. (15-6): $B = 0.25(12 - 5)^{2/3} = 0.9148$
$A = 50 + 56(1 - 0.9148) = 54.77$

Eq. (15-5): $K_v = \left(\dfrac{54.77 + \sqrt{628.3}}{54.77}\right)^{0.9148} = 1.412$

Eq. (15-10): $K_s = 0.4867 + 0.2132/10 = 0.508$

Eq. (15-11): $K_m = 1.25 + 0.0036(0.71)^2 = 1.252$, where $K_{mb} = 1.25$

Eq. (15-15): $(K_L)_P = 1.6831(10^9)^{-0.0323} = 0.862$
$(K_L)_G = 1.6831(10^9/2)^{-0.0323} = 0.881$

Eq. (15-14): $(C_L)_P = 3.4822(10^9)^{-0.0602} = 1.000$
$(C_L)_G = 3.4822(10^9/2)^{-0.0602} = 1.043$

Analyze for 10^9 pinion cycles at 0.999 reliability.

Eq. (15-19): $K_R = 0.50 - 0.25 \log(1 - 0.999) = 1.25$
$C_R = \sqrt{K_R} = \sqrt{1.25} = 1.118$

Bending
Pinion:
Eq. (15-23): $(s_{at})_P = 44(300) + 2100 = 15\ 300$ psi

Eq. (15-4): $(s_{wt})_P = \dfrac{15\ 300(0.862)}{1(1)(1.25)} = 10\ 551$ psi

Eq. (15-3): $W^t = \dfrac{(s_{wt})_P F K_x J_P}{P_d K_o K_v K_s K_m}$

$$= \frac{10\ 551(0.71)(1)(0.241)}{10(1)(1.412)(0.508)(1.252)} = 201 \text{ lbf}$$

$$H_1 = \frac{201(628.3)}{33\ 000} = 3.8 \text{ hp}$$

Gear: $(s_{at})_G = 15\ 300$ psi

Eq. (15-4): $(s_{wt})_G = \dfrac{15\ 300(0.881)}{1(1)(1.25)} = 10\ 783$ psi

Eq. (15-3): $W^t = \dfrac{10\ 783(0.71)(1)(0.201)}{10(1)(1.412)(0.508)(1.252)} = 171.4$ lbf

$$H_2 = \frac{171.4(628.3)}{33\ 000} = 3.3 \text{ hp}$$

Wear

Pinion:

$$(C_H)_G = 1, \quad I = 0.078, \quad C_p = 2290\sqrt{\text{psi}}, \quad C_{xc} = 2$$

$$C_s = 0.125(0.71) + 0.4375 = 0.526\ 25$$

Eq. (15-22): $(s_{ac})_P = 341(300) + 23\ 620 = 125\ 920$ psi

$$(\sigma_{c,\text{all}})_P = \frac{125\ 920(1)(1)}{1(1)(1.118)} = 112\ 630 \text{ psi}$$

Eq. (15-1): $W^t = \left[\dfrac{(\sigma_{c,\text{all}})_P}{C_p}\right]^2 \dfrac{F d_P I}{K_o K_v K_m C_s C_{xc}}$

$$= \left(\frac{112\ 630}{2290}\right)^2 \left[\frac{0.71(2.000)(0.078)}{1(1.412)(1.252)(0.526\ 25)(2)}\right]$$

$$= 144.0 \text{ lbf}$$

$$H_3 = \frac{144(628.3)}{33\ 000} = 2.7 \text{ hp}$$

Gear:

$$(s_{ac})_G = 125\ 920 \text{ psi}$$

$$(\sigma_{c,\text{all}}) = \frac{125\ 920(1.043)(1)}{1(1)(1.118)} = 117\ 473 \text{ psi}$$

$$W^t = \left(\frac{117\ 473}{2290}\right)^2 \left[\frac{0.71(2.000)(0.078)}{1(1.412)(1.252)(0.526\ 25)(2)}\right] = 156.6 \text{ lbf}$$

$$H_4 = \frac{156.6(628.3)}{33\ 000} = 3.0 \text{ hp}$$

Rating:

$$H = \min(3.8, 3.3, 2.7, 3.0) = 2.7 \text{ hp}$$

Pinion wear controls the power rating. While the basis of the catalog rating is unknown, it is overly optimistic (by a factor of 1.9).

15-11 From Ex. 15-1, the core hardness of both the pinion and gear is 180 Brinell. So $(H_B)_{11}$ and $(H_B)_{21}$ are 180 Brinell and the bending stress numbers are:

$$(s_{at})_P = 44(180) + 2100 = 10\ 020 \text{ psi}$$
$$(s_{at})_G = 10\ 020 \text{ psi}$$

The contact strength of the gear case, based upon the equation derived in Prob. 15-7, is

$$(s_{ac})_G = \frac{C_p}{(C_L)_G C_H}\sqrt{\frac{S_H^2}{S_F}\frac{(s_{at})_P(K_L)_P K_x J_P K_T C_s C_{xc}}{N_P I K_s}}$$

Substituting $(s_{at})_P$ from above and the values of the remaining terms from Ex. 15-1,

$$(s_{ac})_G = \frac{2290}{1.32(1)}\sqrt{\frac{1.5^2}{1.5}\left(\frac{10\ 020(1)(1)(0.216)(1)(0.575)(2)}{25(0.065)(0.529)}\right)}$$
$$= 114\ 331 \text{ psi}$$
$$(H_B)_{22} = \frac{114\ 331 - 23\ 620}{341} = 266 \text{ Brinell}$$

The pinion contact strength is found using the relation from Prob. 15-7:

$$(s_{ac})_P = (s_{ac})_G m_G^{0.0602} C_H = 114\ 331(1)^{0.0602}(1) = 114\ 331 \text{ psi}$$
$$(H_B)_{12} = \frac{114\ 331 - 23\ 600}{341} = 266 \text{ Brinell}$$

	Core	Case
Pinion	180	266
Gear	180	266

Realization of hardnesses

The response of students to this part of the question would be a function of the extent to which heat-treatment procedures were covered in their materials and manufacturing prerequisites, and how quantitative it was. The most important thing is to have the student think about it.

The instructor can comment in class when students' curiosity is heightened. Options that will surface may include:

(a) Select a through-hardening steel which will meet or exceed core hardness in the hot-rolled condition, then heat-treating to gain the additional 86 points of Brinell hardness by bath-quenching, then tempering, then generating the teeth in the blank.

(b) Flame or induction hardening are possibilities.

(c) The hardness goal for the case is sufficiently modest that carburizing and case hardening may be too costly. In this case the material selection will be different.

(d)The initial step in a nitriding process brings the core hardness to 33–38 Rockwell C-scale (about 300–350 Brinell), which is too much.

15-12 Computer programs will vary.

15-13 A design program would ask the user to make the a priori decisions, as indicated in Sec. 15-5, p. 806, of the text. The decision set can be organized as follows:

A priori decisions:
- Function: H, K_o, rpm, m_G, temp., N_L, R
- Design factor: n_d $(S_F = n_d\ ,\ S_H = \sqrt{n_d}\ )$
- Tooth system: Involute, Straight Teeth, Crowning, ϕ_n
- Straddling: K_{mb}
- Tooth count: N_P $(N_G = m_G N_P)$

Design decisions:
- Pitch and Face: P_d , F
- Quality number: Q_v
- Pinion hardness: $(H_B)_1$, $(H_B)_3$
- Gear hardness: $(H_B)_2$, $(H_B)_4$

First, gather all of the equations one needs, then arrange them before coding. Find the required hardnesses, express the consequences of the chosen hardnesses, and allow for revisions as appropriate.

	Pinion Bending	Gear Bending	Pinion Wear	Gear Wear
Load-induced stress (Allowable stress)	$s_t = \dfrac{W^t P K_o K_v K_m K_s}{F K_x J_P} = s_{11}$	$s_t = \dfrac{W^t P K_o K_v K_m K_s}{F K_x J_G} = s_{21}$	$\sigma_c = C_p \left(\dfrac{W^t K_o K_v C_s C_{xc}}{F d_P I} \right)^{1/2} = s_{12}$	$s_{22} = s_{12}$
Tabulated strength	$(s_{at})_P = \dfrac{s_{11} S_F K_T K_R}{(K_L)_P}$	$(s_{at})_G = \dfrac{s_{21} S_F K_T K_R}{(K_L)_G}$	$(s_{ac})_P = \dfrac{s_{12} S_H K_T C_R}{(C_L)_P (C_H)_P}$	$(s_{ac})_G = \dfrac{s_{22} S_H K_T C_R}{(C_L)_G (C_H)_G}$
Associated hardness	$\text{Bhn} = \begin{cases} \dfrac{(s_{at})_P - 2100}{44} \\ \dfrac{(s_{at})_P - 5980}{48} \end{cases}$	$\text{Bhn} = \begin{cases} \dfrac{(s_{at})_G - 2100}{44} \\ \dfrac{(s_{at})_G - 5980}{48} \end{cases}$	$\text{Bhn} = \begin{cases} \dfrac{(s_{ac})_P - 23\,620}{341} \\ \dfrac{(s_{ac})_P - 29\,560}{363.6} \end{cases}$	$\text{Bhn} = \begin{cases} \dfrac{(s_{ac})_P - 23\,620}{341} \\ \dfrac{(s_{ac})_P - 29\,560}{363.6} \end{cases}$
Chosen hardness	$(H_B)_{11}$	$(H_B)_{21}$	$(H_B)_{12}$	$(H_B)_{22}$
New tabulated strength	$(s_{at1})_P = \begin{cases} 44(H_B)_{11} + 2100 \\ 48(H_B)_{11} + 5980 \end{cases}$	$(s_{at1})_G = \begin{cases} 44(H_B)_{21} + 2100 \\ 48(H_B)_{21} + 5980 \end{cases}$	$(s_{ac1})_P = \begin{cases} 341(H_B)_{12} + 23\,620 \\ 363.6(H_B)_{12} + 29\,560 \end{cases}$	$(s_{ac1})_G = \begin{cases} 341(H_B)_{22} + 23\,620 \\ 363.6(H_B)_{22} + 29\,560 \end{cases}$
Factor of safety	$n_{11} = \dfrac{\sigma_{\text{all}}}{\sigma} = \dfrac{(s_{at1})_P (K_L)_P}{s_{11} K_T K_R}$	$n_{21} = \dfrac{(s_{at1})_G (K_L)_G}{s_{21} K_T K_R}$	$n_{12} = \left[\dfrac{(s_{ac1})_P (C_L)_P (C_H)_P}{s_{12} K_T C_R} \right]^2$	$n_{22} = \left[\dfrac{(s_{ac1})_G (C_L)_G (C_H)_G}{s_{22} K_T C_R} \right]^2$

Note: $S_F = n_d, \quad S_H = \sqrt{S_F}$

15-14 $N_W = 1$, $N_G = 56$, $P_t = 8$ teeth/in, $d = 1.5$ in, $H_o = 1$hp, $\phi_n = 20°$, $t_a = 70°$F, $K_a = 1.25$, $n_d = 1$, $F_e = 2$ in, $A = 850$ in^2

(a) $m_G = N_G/N_W = 56$, $d_G = N_G/P_t = 56/8 = 7.0$ in

$p_x = \pi / 8 = 0.3927$ in, $C = 1.5 + 7 = 8.5$ in

Eq. (15-39): $a = p_x / \pi = 0.3927 / \pi = 0.125$ in

Eq. (15-40): $b = 0.3683\, p_x = 0.1446$ in

Eq. (15-41): $h_t = 0.6866\, p_x = 0.2696$ in

Eq. (15-42): $d_o = 1.5 + 2(0.125) = 1.75$ in

Eq. (15-43): $d_r = 3 - 2(0.1446) = 2.711$ in

Eq. (15-44): $D_t = 7 + 2(0.125) = 7.25$ in

Eq. (15-45): $D_r = 7 - 2(0.1446) = 6.711$ in

Eq. (15-46): $c = 0.1446 - 0.125 = 0.0196$ in

Eq. (15-47): $(F_W)_{\max} = 2\sqrt{2(7)(0.125)} = 2.646$ in

$$V_W = \pi(1.5)(1725/12) = 677.4 \text{ ft/min}$$

$$V_G = \frac{\pi(7)(1725/56)}{12} = 56.45 \text{ ft/min}$$

Eq. (13-27): $L = p_x N_W = 0.3927$ in

Eq. (13-28): $\lambda = \tan^{-1}\left(\dfrac{0.3927}{\pi(1.5)}\right) = 4.764^\circ$

$$P_n = \frac{P_t}{\cos\lambda} = \frac{8}{\cos 4.764^\circ} = 8.028$$

$$p_n = \frac{\pi}{P_n} = 0.3913 \text{ in}$$

Eq. (15-62): $V_s = \dfrac{\pi(1.5)(1725)}{12\cos 4.764^\circ} = 679.8$ ft/min

(b)

Eq. (15-38): $f = 0.103\exp\left[-0.110(679.8)^{0.450}\right] + 0.012 = 0.0250$

Eq. (15-54):

$$e = \frac{\cos\phi_n - f\tan\lambda}{\cos\phi_n + f\cot\lambda} = \frac{\cos 20^\circ - 0.0250\tan 4.764^\circ}{\cos 20^\circ + 0.0250\cot 4.764^\circ} = 0.7563 \quad \textit{Ans.}$$

Eq. (15-58): $W_G^t = \frac{33\,000 n_d H_o K_a}{V_G e} = \frac{33\,000(1)(1)(1.25)}{56.45(0.7563)} = 966 \text{ lbf}$ *Ans.*

Eq. (15-57): $W_W^t = W_G^t \frac{\cos\phi_n \sin\lambda + f\cos\lambda}{\cos\phi_n \cos\lambda - f\sin\lambda}$

$$= 966\left(\frac{\cos 20^\circ \sin 4.764^\circ + 0.025\cos 4.764^\circ}{\cos 20^\circ \cos 4.764^\circ - 0.025\sin 4.764^\circ}\right)$$

$= 106.4$ lbf *Ans.*

(c)

Eq. (15-33): $C_s = 1190 - 477 \log 7.0 = 787$

Eq. (15-36): $C_m = 0.0107\sqrt{-56^2 + 56(56) + 5145} = 0.767$

Eq. (15-37): $C_v = 0.659\exp[-0.0011(679.8)] = 0.312$

Eq. (15-38): $(W^t)_{\text{all}} = 787(7)^{0.8}(2)(0.767)(0.312) = 1787$ lbf

Since $W_G^t < (W^t)_{\text{all}}$, the mesh will survive at least 25 000 h.

Eq. (15-61): $W_f = \frac{0.025(966)}{0.025\sin 4.764^\circ - \cos 20^\circ \cos 4.764^\circ} = -29.5 \text{ lbf}$

Eq. (15-63): $H_f = \frac{29.5(679.8)}{33\,000} = 0.608 \text{ hp}$

$$H_W = \frac{106.4(677.4)}{33\,000} = 2.18 \text{ hp}$$

$$H_G = \frac{966(56.45)}{33\,000} = 1.65 \text{ hp}$$

The mesh is sufficient *Ans.*

$$P_n = P_t / \cos\lambda = 8/\cos 4.764^\circ = 8.028$$

$$p_n = \pi / 8.028 = 0.3913 \text{ in}$$

$$\sigma_G = \frac{966}{0.3913(0.5)(0.125)} = 39\,500 \text{ psi}$$

The stress is high. At the rated horsepower,

$$\sigma_G = \frac{1}{1.65} 39\,500 = 23\,940 \text{ psi} \quad \text{acceptable}$$

(d)

Eq. (15-52): $A_{\text{min}} = 43.2(8.5)^{1.7} = 1642 \text{ in}^2 < 1700 \text{ in}^2$

Eq. (15-49): $H_{\text{loss}} = 33\,000(1 - 0.7563)(2.18) = 17\,530 \text{ ft} \cdot \text{lbf/min}$

Assuming a fan exists on the worm shaft,

Eq. (15-50): $\hbar_{CR} = \dfrac{1725}{3939} + 0.13 = 0.568 \text{ ft} \cdot \text{lbf/(min} \cdot \text{in}^2 \cdot {}^{\circ}\text{F)}$

Eq. (15-51): $t_s = 70 + \dfrac{17\,530}{0.568(1700)} = 88.2^{\circ}\text{F}$ *Ans.*

15-15 Problem statement values of 25 hp, 1125 rev/min, $m_G = 10$, $K_a = 1.25$, $n_d = 1.1$, $\phi_n = 20°$, $t_a = 70°F$ are not referenced in the table. The first four parameters listed in the table were selected as design decisions.

	15-15	15-16	15-17	15-18	15-19	15-20	15-21	15-22
p_x	1.75	1.75	1.75	1.75	1.75	1.75	1.75	1.75
d_W	3.60	3.60	3.60	3.60	3.60	4.10	3.60	3.60
F_G	2.40	1.68	1.43	1.69	2.40	2.25	2.4	2.4
A	2000	2000	2000	2000	2000	2000	2500	2600
							FAN	FAN
H_W	38.2	38.2	38.2	38.2	38.2	38.0	41.2	41.2
H_G	36.2	36.2	36.2	36.2	36.2	36.1	37.7	37.7
H_f	1.87	1.47	1.97	1.97	1.97	1.85	3.59	3.59
N_W	3	3	3	3	3	3	3	3
N_G	30	30	30	30	30	30	30	30
K_W				125	80	50	115	185
C_s	607	854	1000					
C_m	0.759	0.759	0.759					
C_v	0.236	0.236	0.236					
V_G	492	492	492	492	492	563	492	492
W_G^t	2430	2430	2430	2430	2430	2120	2524	2524
W_W^t	1189	1189	1189	1189	1189	1038	1284	1284
f	0.0193	0.0193	0.0193	0.0193	0.0193	0.0183	0.034	0.034
e	0.948	0.948	0.948	0.948	0.948	0.951	0.913	0.913
$(P_t)_G$	1.795	1.795	1.795	1.795	1.795	1.571	1.795	1.795
P_n	1.979	1.979	1.979	1.979	1.979	1.732	1.979	1.979
C-to-C	10.156	10.156	10.156	10.156	10.156	11.6	10.156	10.156
t_s	177	177	177	177	177	171	179.6	179.6
L	5.25	5.25	5.25	5.25	5.25	6.0	5.25	5.25
λ	24.9	24.9	24.9	24.9	24.9	24.98	24.9	24.9
σ_G	5103	7290	8565	7247	5103	4158	5301	5301
d_G	16.71	16.71	16.71	16.71	16.71	19.099	16.7	16.71