## Chapter 11

11-1 For the deep-groove 02-series ball bearing with $R=0.90$, the design life $x_{D}$, in multiples of rating life, is

$$
x_{D}=\frac{L_{D}}{L_{R}}=\frac{60 \mathcal{L}_{D} n_{D}}{L_{10}}=\frac{60(25000) 350}{10^{6}}=525 \quad \text { Ans. }
$$

The design radial load is

$$
\begin{aligned}
F_{D} & =1.2(2.5)=3.0 \mathrm{kN} \\
\text { Eq. (11-6): } & C_{10}=3.0\left\{\frac{525}{0.02+(4.459-0.02)[\ln (1 / 0.9)]^{1 / 1.483}}\right\}^{1 / 3} \\
& C_{10}=24.3 \mathrm{kN} \quad \text { Ans. }
\end{aligned}
$$

Table 11-2: Choose an 02-35 mm bearing with $C_{10}=25.5 \mathrm{kN} . \quad$ Ans.
Eq. (11-18): $\quad R=\exp \left\{-\left[\frac{525(3 / 25.5)^{3}-0.02}{4.459-0.02}\right]^{1.483}\right\}=0.920 \quad$ Ans.

11-2 For the angular-contact 02-series ball bearing as described, the rating life multiple is

$$
x_{D}=\frac{L_{D}}{L_{R}}=\frac{60 \mathcal{L}_{D} n_{D}}{L_{10}}=\frac{60(40000) 520}{10^{6}}=1248
$$

The design radial load is

$$
F_{D}=1.4(725)=1015 \mathrm{lbf}=4.52 \mathrm{kN}
$$

Eq. (11-6):

$$
\begin{aligned}
C_{10} & =1015\left\{\frac{1248}{0.02+(4.459-0.02)[\ln (1 / 0.9)]^{1 / 1.483}}\right\}^{1 / 3} \\
& =10930 \mathrm{lbf}=48.6 \mathrm{kN}
\end{aligned}
$$

Table 11-2: $\quad$ Select an 02-60 mm bearing with $C_{10}=55.9 \mathrm{kN}$. Ans.
Eq. (11-18): $\quad R=\exp \left\{-\left[\frac{1248(4.52 / 55.9)^{3}-0.02}{4.439}\right]^{1.483}\right\}=0.945 \quad$ Ans.

11-3 For the straight-roller 03-series bearing selection, $x_{D}=1248$ rating lives from Prob. 11-2 solution.

$$
\begin{aligned}
& F_{D}=1.4(2235)=3129 \mathrm{lbf}=13.92 \mathrm{kN} \\
& C_{10}=13.92\left(\frac{1248}{1}\right)^{3 / 10}=118 \mathrm{kN}
\end{aligned}
$$

Table 11-3: $\quad$ Select an $03-60 \mathrm{~mm}$ bearing with $C_{10}=123 \mathrm{kN}$. Ans.
Eq. (11-18): $\quad R=\exp \left\{-\left[\frac{1248(13.92 / 123)^{10 / 3}-0.02}{4.459-0.02}\right]^{1.483}\right\}=0.917 \quad$ Ans.

11-4 The combined reliability of the two bearings selected in Probs. 11-2 and 11-3 is

$$
R=(0.945)(0.917)=0.867 \quad \text { Ans. }
$$

We can choose a reliability goal of $\sqrt{0.90}=0.95$ for each bearing. We make the selections, find the existing reliabilities, multiply them together, and observe that the reliability goal is exceeded due to the roundup of capacity upon table entry.

Another possibility is to use the reliability of one bearing, say $R_{1}$. Then set the reliability goal of the second as

$$
R_{2}=\frac{0.90}{R_{1}}
$$

or vice versa. This gives three pairs of selections to compare in terms of cost, geometry implications, etc.

11-5 Establish a reliability goal of $\sqrt{0.90}=0.95$ for each bearing. For an 02-series angular contact ball bearing,

$$
\begin{aligned}
C_{10} & =1015\left\{\frac{1248}{0.02+4.439[\ln (1 / 0.95)]^{1 / .483}}\right\}^{1 / 3} \\
& =12822 \mathrm{lbf}=57.1 \mathrm{kN}
\end{aligned}
$$

Select an 02-65 mm angular-contact bearing with $C_{10}=63.7 \mathrm{kN}$.

$$
R_{A}=\exp \left\{-\left[\frac{1248(4.52 / 63.7)^{3}-0.02}{4.439}\right]^{1.483}\right\}=0.962
$$

For an 03-series straight roller bearing,

$$
C_{10}=13.92\left\{\frac{1248}{0.02+4.439[\ln (1 / 0.95)]^{1 / .483}}\right\}^{3 / 10}=136.5 \mathrm{kN}
$$

Select an 03-65 mm straight-roller bearing with $C_{10}=138 \mathrm{kN}$.

$$
R_{B}=\exp \left\{-\left[\frac{1248(13.92 / 138)^{10 / 3}-0.02}{4.439}\right]^{1.483}\right\}=0.953
$$

The overall reliability is $R=(0.962)(0.953)=0.917$, which exceeds the goal.

11-6 For the straight cylindrical roller bearing specified with a service factor of $1, R=0.95$ and $F_{R}=20 \mathrm{kN}$.

$$
\begin{aligned}
& x_{D}=\frac{L_{D}}{L_{R}}=\frac{60 \mathcal{L}_{D} n_{D}}{L_{10}}=\frac{60(8000) 950}{10^{6}}=456 \\
& C_{10}=20\left\{\frac{456}{0.02+4.439[\ln (1 / 0.95)]^{1 / 1.483}}\right\}^{3 / 10}=145 \mathrm{kN} \quad \text { Ans. }
\end{aligned}
$$

11-7 Both bearings need to be rated in terms of the same catalog rating system in order to compare them. Using a rating life of one million revolutions, both bearings can be rated in terms of a Basic Load Rating.

Eq. (11-3): $\quad \begin{aligned} C_{A} & =F_{A}\left(\frac{L_{A}}{L_{R}}\right)^{1 / a}=F_{A}\left(\frac{\mathcal{L}_{A} n_{A} 60}{L_{R}}\right)^{1 / a}=2.0\left[\frac{(3000)(500)(60)}{10^{6}}\right]^{1 / 3} \\ & =8.96 \mathrm{kN}\end{aligned}$
Bearing $B$ already is rated at one million revolutions, so $C_{B}=7.0 \mathrm{kN}$. Since $C_{A}>C_{B}$, bearing $A$ can carry the larger load.
Ans.

11-8 $\quad F_{D}=2 \mathrm{kN}, L_{D}=10^{9} \mathrm{rev}, R=0.90$
Eq. (11-3): $\quad C_{10}=F_{D}\left(\frac{L_{D}}{L_{R}}\right)^{1 / a}=2\left(\frac{10^{9}}{10^{6}}\right)^{1 / 3}=20 \mathrm{kN}$ Ans.

11-9 $\quad F_{D}=800 \mathrm{lbf}, \mathcal{L}_{D}=12000$ hours, $n_{D}=350 \mathrm{rev} / \mathrm{min}, R=0.90$

Eq. (11-3): $\quad C_{10}=F_{D}\left(\frac{\mathcal{L}_{D} n_{D} 60}{L_{R}}\right)^{1 / a}=800\left(\frac{12000(350)(60)}{10^{6}}\right)^{1 / 3}=5050 \mathrm{lbf} \quad$ Ans

11-10 $F_{D}=4 \mathrm{kN}, \mathcal{L}_{D}=8000$ hours, $n_{D}=500 \mathrm{rev} / \mathrm{min}, R=0.90$
Eq. (11-3): $\quad C_{10}=F_{D}\left(\frac{\mathcal{L}_{D} n_{D} 60}{L_{R}}\right)^{1 / a}=4\left(\frac{8000(500)(60)}{10^{6}}\right)^{1 / 3}=24.9 \mathrm{kN} \quad$ Ans

11-11 $F_{D}=650 \mathrm{lbf}, n_{D}=400 \mathrm{rev} / \mathrm{min}, R=0.95$

$$
\mathcal{L}_{D}=(5 \text { years })(40 \mathrm{~h} / \text { week })(52 \text { week } / \text { year })=10400 \text { hours }
$$

Assume an application factor of one. The multiple of rating life is

$$
x_{D}=\frac{L_{D}}{L_{R}}=\frac{(10400)(400)(60)}{10^{6}}=249.6
$$

Eq. (11-6): $\quad C_{10}=(1)(650)\left\{\frac{249.6}{0.02+4.439[\ln (1 / 0.95)]^{1 / 1.483}}\right\}^{1 / 3}$

$$
=4800 \mathrm{lbf} \quad \text { Ans. }
$$

11-12 $F_{D}=9 \mathrm{kN}, L_{D}=10^{8} \mathrm{rev}, R=0.99$
Assume an application factor of one. The multiple of rating life is

$$
\begin{aligned}
x_{D} & =\frac{L_{D}}{L_{R}}=\frac{10^{8}}{10^{6}}=100 \\
C_{10} & =(1)(9)\left\{\frac{100}{0.02+4.439[\ln (1 / 0.99)]^{1 / 1.483}}\right\}^{1 / 3} \\
& =69.2 \mathrm{kN} \quad \text { Ans. }
\end{aligned}
$$

11-13 $F_{D}=11 \mathrm{kips}, \mathcal{L}_{D}=20000$ hours, $n_{D}=200 \mathrm{rev} / \mathrm{min}, R=0.99$
Assume an application factor of one. Use the Weibull parameters for Manufacturer 2 on p. 608.

The multiple of rating life is

$$
\begin{aligned}
x_{D}= & \frac{L_{D}}{L_{R}}=\frac{(20000)(200)(60)}{10^{6}}=240 \\
C_{10} & =(1)(11)\left\{\frac{240}{0.02+4.439[\ln (1 / 0.99)]^{1 / .483}}\right\}^{1 / 3} \\
& =113 \mathrm{kips} \text { Ans. }
\end{aligned}
$$

Eq. (11-6):

11-14 From the solution to Prob. 3-68, the ground reaction force carried by the bearing at $C$ is $R_{C}=F_{D}=178 \mathrm{lbf}$. Use the Weibull parameters for Manufacturer 2 on p. 608.

$$
x_{D}=\frac{L_{D}}{L_{R}}=\frac{15000(1200)(60)}{10^{6}}=1080
$$

Eq. (11-7):

$$
\begin{aligned}
C_{10} & =a_{f} F_{D}\left[\frac{x_{D}}{x_{0}+\left(\theta-x_{0}\right)\left(1-R_{D}\right)^{1 / b}}\right]^{1 / a} \\
C_{10} & =1.2(178)\left[\frac{1080}{0.02+(4.459-0.02)(1-0.95)^{1 / 1.483}}\right]^{1 / 3} \\
& =2590 \mathrm{lbf} \quad \text { Ans. }
\end{aligned}
$$

11-15 From the solution to Prob. 3-69, the ground reaction force carried by the bearing at $C$ is $R_{C}=F_{D}=1.794 \mathrm{kN}$. Use the Weibull parameters for Manufacturer 2 on p. 608.

$$
x_{D}=\frac{L_{D}}{L_{R}}=\frac{15000(1200)(60)}{10^{6}}=1080
$$

Eq. (11-7): $\quad C_{10}=a_{f} F_{D}\left[\frac{x_{D}}{x_{0}+\left(\theta-x_{0}\right)\left(1-R_{D}\right)^{1 / b}}\right]^{1 / a}$

$$
\begin{aligned}
C_{10} & =1.2(1.794)\left[\frac{1080}{0.02+(4.459-0.02)(1-0.95)^{1 / .483}}\right]^{1 / 3} \\
& =26.1 \mathrm{kN} \quad \text { Ans. }
\end{aligned}
$$

11-16 From the solution to Prob. 3-70, $R_{C z}=-327.99 \mathrm{lbf}, R_{C y}=-127.27 \mathrm{lbf}$

$$
R_{C}=F_{D}=\left[(-327.99)^{2}+(-127.27)^{2}\right]^{1 / 2}=351.8 \mathrm{lbf}
$$

Use the Weibull parameters for Manufacturer 2 on p. 608.

$$
x_{D}=\frac{L_{D}}{L_{R}}=\frac{15000(1200)(60)}{10^{6}}=1080
$$

Eq. (11-7):

$$
\begin{aligned}
C_{10} & =a_{f} F_{D}\left[\frac{x_{D}}{x_{o}+\left(\theta-x_{o}\right)\left(1-R_{D}\right)^{1 / b}}\right]^{1 / a} \\
C_{10} & =1.2(351.8)\left[\frac{1080}{0.02+(4.459-0.02)(1-0.95)^{1 / 1.483}}\right]^{1 / 3} \\
& =5110 \mathrm{lbf} \quad \text { Ans. }
\end{aligned}
$$

11-17 From the solution to Prob. 3-71, $R_{C z}=-150.7 \mathrm{~N}, R_{C y}=-86.10 \mathrm{~N}$

$$
R_{C}=F_{D}=\left[(-150.7)^{2}+(-86.10)^{2}\right]^{1 / 2}=173.6 \mathrm{~N}
$$

Use the Weibull parameters for Manufacturer 2 on p. 608.

$$
x_{D}=\frac{L_{D}}{L_{R}}=\frac{15000(1200)(60)}{10^{6}}=1080
$$

Eq. (11-7): $\quad C_{10}=a_{f} F_{D}\left[\frac{x_{D}}{x_{0}+\left(\theta-x_{0}\right)\left(1-R_{D}\right)^{1 / b}}\right]^{1 / a}$

$$
\begin{aligned}
C_{10} & =1.2(173.6)\left[\frac{1080}{0.02+(4.459-0.02)(1-0.95)^{1 / .483}}\right]^{1 / 3} \\
& =2520 \mathrm{~N} \text { Ans. }
\end{aligned}
$$

11-18 From the solution to Prob. 3-77, $R_{A z}=444 \mathrm{~N}, R_{A y}=2384 \mathrm{~N}$

$$
R_{A}=F_{D}=\left(444^{2}+2384^{2}\right)^{1 / 2}=2425 \mathrm{~N}=2.425 \mathrm{kN}
$$

Use the Weibull parameters for Manufacturer 2 on p. 608. The design speed is equal to the speed of shaft $A D$,

$$
\begin{aligned}
& n_{D}=\frac{d_{F}}{d_{C}} n_{i}=\frac{125}{250}(191)=95.5 \mathrm{rev} / \mathrm{min} \\
& x_{D}=\frac{L_{D}}{L_{R}}=\frac{12000(95.5)(60)}{10^{6}}=68.76
\end{aligned}
$$

Eq. (11-7):

$$
C_{10}=a_{f} F_{D}\left[\frac{x_{D}}{x_{0}+\left(\theta-x_{0}\right)\left(1-R_{D}\right)^{1 / b}}\right]^{1 / a}
$$

$$
\begin{aligned}
C_{10} & =(1)(2.425)\left[\frac{68.76}{0.02+(4.459-0.02)(1-0.95)^{1 / 1.483}}\right]^{1 / 3} \\
& =11.7 \mathrm{kN} \quad \text { Ans. }
\end{aligned}
$$

11-19 From the solution to Prob. $3-79, R_{A z}=54.0 \mathrm{lbf}, R_{A y}=140 \mathrm{lbf}$

$$
R_{A}=F_{D}=\left(54.0^{2}+140^{2}\right)^{1 / 2}=150.1 \mathrm{lbf}
$$

Use the Weibull parameters for Manufacturer 2 on p. 608. The design speed is equal to the speed of shaft $A D$,

$$
\begin{aligned}
& n_{D}=\frac{d_{F}}{d_{C}} n_{i}=\frac{10}{5}(280)=560 \mathrm{rev} / \mathrm{min} \\
& x_{D}=\frac{L_{D}}{L_{R}}=\frac{14000(560)(60)}{10^{6}}=470.4
\end{aligned}
$$

Eq. (11-7): $\quad C_{10}=a_{f} F_{D}\left[\frac{x_{D}}{x_{0}+\left(\theta-x_{0}\right)\left(1-R_{D}\right)^{1 / b}}\right]^{1 / a}$

$$
\begin{aligned}
C_{10} & =(1)(150.1)\left[\frac{470.4}{0.02+(4.459-0.02)(1-0.98)^{1 / 1.483}}\right]^{3 / 10} \\
& =1320 \mathrm{lbf} \text { Ans. }
\end{aligned}
$$

11-20 (a) $F_{a}=3 \mathrm{kN}, F_{r}=7 \mathrm{kN}, n_{D}=500 \mathrm{rev} / \mathrm{min}, V=1.2$
From Table 11-2, with a 65 mm bore, $C_{0}=34.0 \mathrm{kN}$.

$$
F_{a} / C_{0}=3 / 34=0.088
$$

From Table 11-1, $0.28 \leq e \leq 3.0$.

$$
\frac{F_{a}}{V F_{r}}=\frac{3}{(1.2)(7)}=0.357
$$

Since this is greater than $e$, interpolating Table 11-1 with $F_{a} / C_{0}=0.088$, we obtain $X_{2}=0.56$ and $Y_{2}=1.53$.

Eq. (11-9): $\quad F_{e}=X_{i} V F_{r}+Y_{i} F_{a}=(0.56)(1.2)(7)+(1.53)(3)=9.29 \mathrm{kN}$ Ans.

$$
F_{e}>F_{r} \text { so use } F_{e} .
$$

(b) Use Eq. (11-7) to determine the necessary rated load the bearing should have to carry the equivalent radial load for the desired life and reliability. Use the Weibull parameters for Manufacturer 2 on p. 608.

$$
x_{D}=\frac{L_{D}}{L_{R}}=\frac{10000(500)(60)}{10^{6}}=300
$$

Eq. (11-7): $C_{10}=a_{f} F_{D}\left[\frac{x_{D}}{x_{0}+\left(\theta-x_{0}\right)\left(1-R_{D}\right)^{1 / b}}\right]^{1 / a}$

$$
\begin{aligned}
C_{10} & =(1)(9.29)\left[\frac{300}{0.02+(4.459-0.02)(1-0.95)^{1 / 1.483}}\right]^{1 / 3} \\
& =73.4 \mathrm{kN}
\end{aligned}
$$

From Table 11-2, the 65 mm bearing is rated for 55.9 kN , which is less than the necessary rating to meet the specifications. This bearing should not be expected to meet the load, life, and reliability goals.

Ans.

11-21 (a) $F_{a}=2 \mathrm{kN}, F_{r}=5 \mathrm{kN}, n_{D}=400 \mathrm{rev} / \mathrm{min}, V=1$
From Table 11-2, 30 mm bore, $C_{10}=19.5 \mathrm{kN}, C_{0}=10.0 \mathrm{kN}$

$$
F_{a} / C_{0}=2 / 10=0.2
$$

From Table 11-1, $0.34 \leq e \leq 0.38$.

$$
\frac{F_{a}}{V F_{r}}=\frac{2}{(1)(5)}=0.4
$$

Since this is greater than $e$, interpolating Table 11-1, with $F_{a} / C_{0}=0.2$, we obtain $X_{2}=$ 0.56 and $Y_{2}=1.27$.

Eq. (11-9): $F_{e}=X_{i} V F_{r}+Y_{i} F_{a}=(0.56)(1)(5)+(1.27)(2)=5.34 \mathrm{kN} \quad$ Ans. $F_{e}>F_{r}$ so use $F_{e}$.
(b) Solve Eq. (11-7) for $x_{D}$.

$$
\begin{aligned}
& x_{D}=\left(\frac{C_{10}}{a_{f} F_{D}}\right)^{a}\left[x_{0}+\left(\theta-x_{0}\right)\left(1-R_{D}\right)^{1 / b}\right] \\
& x_{D}=\left(\frac{19.5}{(1)(5.34)}\right)^{3}\left[0.02+(4.459-0.02)(1-0.99)^{1 / 1.483}\right] \\
& x_{D}=10.66 \\
& x_{D}=\frac{L_{D}}{L_{R}}=\frac{\mathcal{L}_{D} n_{D}(60)}{10^{6}}
\end{aligned}
$$

$$
\mathcal{L}_{D}=\frac{x_{D}\left(10^{6}\right)}{n_{D}(60)}=\frac{10.66\left(10^{6}\right)}{(400)(60)}=444 \mathrm{~h} \quad \text { Ans. }
$$

11-22 $F_{r}=8 \mathrm{kN}, R=0.9, L_{D}=10^{9} \mathrm{rev}$
Eq. (11-3): $\quad C_{10}=F_{D}\left(\frac{L_{D}}{L_{R}}\right)^{1 / a}=8\left(\frac{10^{9}}{10^{6}}\right)^{1 / 3}=80 \mathrm{kN}$

From Table 11-2, select the 85 mm bore. Ans.

11-23 $F_{r}=8 \mathrm{kN}, \quad F_{a}=2 \mathrm{kN}, \quad V=1, \quad R=0.99$
Use the Weibull parameters for Manufacturer 2 on p. 608.

$$
x_{D}=\frac{L_{D}}{L_{R}}=\frac{10000(400)(60)}{10^{6}}=240
$$

First guess: Choose from middle of Table 11-1, $X=0.56, Y=1.63$

Eq. (11-9): $\quad F_{e}=0.56(1)(8)+1.63(2)=7.74 \mathrm{kN}$

$$
F_{e}<F_{r} \text {, so just use } F_{r} \text { as the design load. }
$$

Eq. (11-7): $\quad C_{10}=a_{f} F_{D}\left[\frac{x_{D}}{x_{o}+\left(\theta-x_{o}\right)\left(1-R_{D}\right)^{1 / b}}\right]^{1 / a}$

$$
C_{10}=(1)(8)\left[\frac{240}{0.02+(4.459-0.02)(1-0.99)^{1 / 1.483}}\right]^{1 / 3}=82.5 \mathrm{kN}
$$

From Table 11-2, try 85 mm bore with $C_{10}=83.2 \mathrm{kN}, C_{0}=53.0 \mathrm{kN}$ Iterate the previous process:

$$
F_{a} / C_{0}=2 / 53=0.038
$$

Table 11-1: $\quad 0.22 \leq e \leq 0.24$

$$
\frac{F_{a}}{V F_{r}}=\frac{2}{1(8)}=0.25>e
$$

Interpolate Table 11-1 with $F_{a} / C_{0}=0.038$ to obtain $X_{2}=0.56$ and $Y_{2}=1.89$.
Eq. (11-9): $\quad F_{e}=0.56(1) 8+1.89(2)=8.26>F_{r}$
Eq. (11-7): $\quad C_{10}=(1)(8.26)\left[\frac{240}{0.02+(4.459-0.02)(1-0.99)^{1 / 1.483}}\right]^{1 / 3}=85.2 \mathrm{kN}$

Table 11-2: Move up to the 90 mm bore with $C_{10}=95.6 \mathrm{kN}, C_{0}=62.0 \mathrm{kN}$. Iterate again:

$$
F_{a} / C_{0}=2 / 62=0.032
$$

Table 11-1: Again, $0.22 \leq e \leq 0.24$

$$
\frac{F_{a}}{V F_{r}}=\frac{2}{1(8)}=0.25>e
$$

Interpolate Table 11-1 with $F_{a} / C_{0}=0.032$ to obtain $X_{2}=0.56$ and $Y_{2}=1.95$.
Eq. (11-9): $\quad F_{e}=0.56(1) 8+1.95(2)=8.38>F_{r}$
Eq. (11-7): $\quad C_{10}=(1)(8.38)\left[\frac{240}{0.02+(4.459-0.02)(1-0.99)^{1 / 1.483}}\right]^{1 / 3}=86.4 \mathrm{kN}$
The 90 mm bore is acceptable. Ans.

11-24 $F_{r}=8 \mathrm{kN}, F_{a}=3 \mathrm{kN}, V=1.2, R=0.9, L_{D}=10^{8} \mathrm{rev}$
First guess: Choose from middle of Table 11-1, $X=0.56, Y=1.63$
Eq. (11-9): $\quad F_{e}=0.56(1.2)(8)+1.63(3)=10.3 \mathrm{kN}$

$$
F_{e}>F_{r}
$$

Eq. (11-3): $\quad C_{10}=F_{e}\left(\frac{L_{D}}{L_{R}}\right)^{1 / a}=10.3\left(\frac{10^{8}}{10^{6}}\right)^{1 / 3}=47.8 \mathrm{kN}$
From Table 11-2, try 60 mm with $C_{10}=47.5 \mathrm{kN}, C_{0}=28.0 \mathrm{kN}$ Iterate the previous process:

$$
F_{a} / C_{0}=3 / 28=0.107
$$

Table 11-1: $\quad 0.28 \leq e \leq 0.30$

$$
\frac{F_{a}}{V F_{r}}=\frac{3}{1.2(8)}=0.313>e
$$

Interpolate Table 11-1 with $F_{a} / C_{0}=0.107$ to obtain $X_{2}=0.56$ and $Y_{2}=1.46$
Eq. (11-9): $\quad F_{e}=0.56(1.2)(8)+1.46(3)=9.76 \mathrm{kN}>F_{r}$
Eq. (11-3): $\quad C_{10}=9.76\left(\frac{10^{8}}{10^{6}}\right)^{1 / 3}=45.3 \mathrm{kN}$
From Table 11-2, we have converged on the 60 mm bearing. Ans.

11-25 $\quad F_{r}=10 \mathrm{kN}, \quad F_{a}=5 \mathrm{kN}, \quad V=1, \quad R=0.95$
Use the Weibull parameters for Manufacturer 2 on p. 608.

$$
x_{D}=\frac{L_{D}}{L_{R}}=\frac{12000(300)(60)}{10^{6}}=216
$$

First guess: Choose from middle of Table 11-1, $X=0.56, Y=1.63$
Eq. (11-9): $\quad F_{e}=0.56(1)(10)+1.63(5)=13.75 \mathrm{kN}$

$$
F_{e}>F_{r} \text {, so use } F_{e} \text { as the design load. }
$$

Eq. (11-7): $\quad C_{10}=a_{f} F_{D}\left[\frac{x_{D}}{x_{0}+\left(\theta-x_{0}\right)\left(1-R_{D}\right)^{1 / b}}\right]^{1 / a}$

$$
C_{10}=(1)(13.75)\left[\frac{216}{0.02+(4.459-0.02)(1-0.95)^{1 / 1.483}}\right]^{1 / 3}=97.4 \mathrm{kN}
$$

From Table 11-2, try 95 mm bore with $C_{10}=108 \mathrm{kN}, C_{0}=69.5 \mathrm{kN}$ Iterate the previous process:

$$
F_{a} / C_{0}=5 / 69.5=0.072
$$

Table 11-1: $\quad 0.27 \leq e \leq 0.28$

$$
\frac{F_{a}}{V F_{r}}=\frac{5}{1(10)}=0.5>e
$$

Interpolate Table 11-1 with $F_{a} / C_{0}=0.072$ to obtain $X_{2}=0.56$ and $Y_{2}=1.62 \doteq 1.63$

Since this is where we started, we will converge back to the same bearing. The 95 mm bore meets the requirements. Ans.

11-26 Note to the Instructor. In the first printing of the 9th edition, the design life was incorrectly given to be $10^{9} \mathrm{rev}$ and will be corrected to $10^{8} \mathrm{rev}$ in subsequent printings. We apologize for the inconvenience.
$F_{r}=9 \mathrm{kN}, \quad F_{a}=3 \mathrm{kN}, \quad V=1.2, \quad R=0.99$
Use the Weibull parameters for Manufacturer 2 on p. 608.

$$
x_{D}=\frac{L_{D}}{L_{R}}=\frac{10^{8}}{10^{6}}=100
$$

First guess: Choose from middle of Table 11-1, $X=0.56, Y=1.63$

Eq. (11-9): $\quad F_{e}=0.56(1.2)(9)+1.63(3)=10.9 \mathrm{kN}$ $F_{e}>F_{r}$, so use $F_{e}$ as the design load.
Eq. (11-7): $\quad C_{10}=a_{f} F_{D}\left[\frac{x_{D}}{x_{0}+\left(\theta-x_{0}\right)\left(1-R_{D}\right)^{1 / b}}\right]^{1 / a}$

$$
C_{10}=(1)(10.9)\left[\frac{100}{0.02+(4.459-0.02)(1-0.99)^{1 / 1.483}}\right]^{1 / 3}=83.9 \mathrm{kN}
$$

From Table 11-2, try 90 mm bore with $C_{10}=95.6 \mathrm{kN}, C_{0}=62.0 \mathrm{kN}$. Try this bearing. Iterate the previous process:

$$
F_{a} / C_{0}=3 / 62=0.048
$$

Table 11-1: $\quad 0.24 \leq e \leq 0.26$

$$
\frac{F_{a}}{V F_{r}}=\frac{3}{1.2(9)}=0.278>e
$$

Interpolate Table 11-1 with $F_{a} / C_{0}=0.048$ to obtain $X_{2}=0.56$ and $Y_{2}=1.79$
Eq. (11-9): $\quad F_{e}=0.56(1.2)(9)+1.79(3)=11.4 \mathrm{kN}>F_{r}$

$$
C_{10}=\frac{11.4}{10.9} 83.9=87.7 \mathrm{kN}
$$

From Table 11-2, this converges back to the same bearing. The 90 mm bore meets the requirements. Ans.

11-27 (a) $n_{D}=1200 \mathrm{rev} / \mathrm{min}, L_{D}=15 \mathrm{kh}, R=0.95, a_{f}=1.2$
From Prob. 3-72, $R_{C y}=183.1 \mathrm{lbf}, R_{C z}=-861.5 \mathrm{lbf}$.

$$
\begin{aligned}
& R_{C}=F_{D}=\left[183.1^{2}+(-861.5)^{2}\right]^{1 / 2}=881 \mathrm{lbf} \\
& x_{D}=\frac{L_{D}}{L_{R}}=\frac{15000(1200)(60)}{10^{6}}=1080
\end{aligned}
$$

Eq. (11-7): $C_{10}=1.2(881)\left[\frac{1080}{0.02+4.439(1-0.95)^{1 / 1.483}}\right]^{1 / 3}$

$$
=12800 \mathrm{lbf}=12.8 \mathrm{kips} \quad \text { Ans. }
$$

(b) Results will vary depending on the specific bearing manufacturer selected. A general engineering components search site such as www.globalspec.com might be useful as a starting point.

11-28 (a) $n_{D}=1200 \mathrm{rev} / \mathrm{min}, L_{D}=15 \mathrm{kh}, R=0.95, a_{f}=1.2$
From Prob. 3-72, $R_{O y}=-208.5 \mathrm{lbf}, R_{O z}=259.3 \mathrm{lbf}$.

$$
\begin{aligned}
& R_{C}=F_{D}=\left[259.3^{2}+(-208.5)^{2}\right]^{1 / 2}=333 \mathrm{lbf} \\
& x_{D}=\frac{L_{D}}{L_{R}}=\frac{15000(1200)(60)}{10^{6}}=1080
\end{aligned}
$$

Eq. (11-7): $C_{10}=1.2(333)\left[\frac{1080}{0.02+4.439(1-0.95)^{1 / 1.483}}\right]^{1 / 3}$

$$
=4837 \mathrm{lbf}=4.84 \mathrm{kips} \quad \text { Ans. }
$$

(b) Results will vary depending on the specific bearing manufacturer selected. A general engineering components search site such as www.globalspec.com might be useful as a starting point.

11-29 (a) $n_{D}=900 \mathrm{rev} / \mathrm{min}, L_{D}=12 \mathrm{kh}, R=0.98, a_{f}=1.2$
From Prob. 3-73, $R_{C y}=8.319 \mathrm{kN}, R_{C z}=-10.830 \mathrm{kN}$.

$$
\begin{aligned}
& R_{C}=F_{D}=\left[8.319^{2}+(-10.830)^{2}\right]^{1 / 2}=13.7 \mathrm{kN} \\
& x_{D}=\frac{L_{D}}{L_{R}}=\frac{12000(900)(60)}{10^{6}}=648
\end{aligned}
$$

Eq. (11-7): $C_{10}=1.2(13.7)\left[\frac{648}{0.02+4.439(1-0.98)^{1 / 1.483}}\right]^{1 / 3}=204 \mathrm{kN} \quad$ Ans.
(b) Results will vary depending on the specific bearing manufacturer selected. A general engineering components search site such as www.globalspec.com might be useful as a starting point.

11-30 (a) $n_{D}=900 \mathrm{rev} / \mathrm{min}, L_{D}=12 \mathrm{kh}, R=0.98, a_{f}=1.2$
From Prob. 3-73, $R_{O y}=5083 \mathrm{~N}, R_{O z}=494 \mathrm{~N}$.

$$
\begin{aligned}
& R_{C}=F_{D}=\left(5083^{2}+494^{2}\right)^{1 / 2}=5106 \mathrm{~N}=5.1 \mathrm{kN} \\
& x_{D}=\frac{L_{D}}{L_{R}}=\frac{12000(900)(60)}{10^{6}}=648
\end{aligned}
$$

Eq. (11-7): $C_{10}=1.2(5.1)\left[\frac{648}{0.02+4.439(1-0.98)^{1 / 1.483}}\right]^{1 / 3}=76.1 \mathrm{kN} \quad$ Ans.
(b) Results will vary depending on the specific bearing manufacturer selected. A general engineering components search site such as www.globalspec.com might be useful as a starting point.

11-31 Assume concentrated forces as shown.

$$
\begin{aligned}
& P_{z}=8(28)=224 \mathrm{lbf} \\
& P_{y}=8(35)=280 \mathrm{lbf} \\
& T=224(2)=448 \mathrm{lbf} \cdot \text { in } \\
& \quad \Sigma T^{x}=-448+1.5 F \cos 20^{\circ}=0 \\
& F=\frac{448}{1.5(0.940)}=318 \mathrm{lbf}
\end{aligned}
$$



$$
\begin{aligned}
& \Sigma M_{O}^{z}= 5.75 P_{y}+11.5 R_{A}^{y}-14.25 F \sin 20^{\circ}=0 \\
& 5.75(280)+11.5 R_{A}^{y}-14.25(318)(0.342)=0 \\
& R_{A}^{y}=-5.24 \mathrm{lbf} \\
& \Sigma M_{O}^{y}=-5.75 P_{z}-11.5 R_{A}^{z}-14.25 F \cos 20^{\circ}=0 \\
&-5.75(224)-11.5 R_{A}^{z}-14.25(318)(0.940)=0 \\
& R_{A}^{z}=-482 \mathrm{lbf} ; \quad R_{A}=\left[(-482)^{2}+(-5.24)^{2}\right]^{1 / 2}=482 \mathrm{lbf} \\
& \Sigma F^{z}= R_{O}^{z}+P_{z}+R_{A}^{z}+F \cos 20^{\circ}=0 \\
& R_{O}^{z}+224-482+318(0.940)=0 \\
& R_{O}^{z}=-40.9 \mathrm{lbf} \\
& \Sigma F^{y}= R_{O}^{y}+P_{y}+R_{A}^{y}-F \sin 20^{\circ}=0 \\
& R_{O}^{y}+ 280-5.24-318(0.342)=0 \\
& R_{O}^{y}=-166 \mathrm{lbf} \\
& R_{O}= {\left[(-40.9)^{2}+(-166)^{2}\right]^{1 / 2}=171 \mathrm{lbf} }
\end{aligned}
$$

So the reaction at $A$ governs.
Reliability Goal: $\sqrt{0.92}=0.96$

$$
\begin{aligned}
F_{D} & =1.2(482)=578 \mathrm{lbf} \\
x_{D} & =35000(350)(60) / 10^{6}=735 \\
C_{10} & =578\left\{\frac{735}{0.02+(4.459-0.02)[\ln (1 / 0.96)]^{1 / 1.483}}\right\}^{1 / 3} \\
& =6431 \mathrm{lbf}=28.6 \mathrm{kN}
\end{aligned}
$$

From Table 11-2, a 40 mm bore angular contact bearing is sufficient with a rating of

11-32 For a combined reliability goal of 0.95 , use $\sqrt{0.95}=0.975$ for the individual bearings.

$$
x_{D}=\frac{40000(420)(60)}{10^{6}}=1008
$$



The resultant of the given forces are

$$
\begin{aligned}
& R_{O}=\left[(-387)^{2}+467^{2}\right]^{1 / 2}=607 \mathrm{lbf} \\
& R_{B}=\left[316^{2}+(-1615)^{2}\right]^{1 / 2}=1646 \mathrm{lbf}
\end{aligned}
$$

At $O$ :

Eq. (11-6):

$$
\begin{aligned}
& C_{10}=1.2(607)\left\{\frac{1008}{0.02+(4.459-0.02)[\ln (1 / 0.975)]^{1 / 1.483}}\right\}^{1 / 3} \\
&=9978 \mathrm{lbf}=44.4 \mathrm{kN}
\end{aligned}
$$

From Table 11-2, select an 02-55 mm angular-contact ball bearing with a basic load rating of 46.2 kN . Ans.

At $B$ :
Eq. (11-6): $\quad C_{10}=1.2(1646)\left\{\frac{1008}{0.02+(4.459-0.02)[\ln (1 / 0.975)]^{1 / 1.483}}\right\}^{3 / 10}$ $=20827 \mathrm{lbf}=92.7 \mathrm{kN}$

From Table 11-3, select an $02-75 \mathrm{~mm}$ or $03-55 \mathrm{~mm}$ cylindrical roller. Ans.

11-33 The reliability of the individual bearings is $R=\sqrt{0.98}=0.9899$

From statics,

$$
\begin{aligned}
& T=(270-50)=\left(P_{1}-P_{2}\right) 125 \\
& \quad=\left(P_{1}-0.15 P_{1}\right) 125 \\
& P_{1}=310.6 \mathrm{~N}, \\
& P_{2}=0.15(310.6)=46.6 \mathrm{~N} \\
& P_{1}+P_{2}=357.2 \mathrm{~N} \\
& F_{A}^{y}=357.2 \sin 45^{\circ}=252.6 \mathrm{~N}=F_{A}^{z}
\end{aligned}
$$



$$
\begin{aligned}
& \sum M_{O}^{z}=850 R_{E}^{y}+300(252.6)=0 \Rightarrow R_{E}^{y}=-89.2 \mathrm{~N} \\
& \sum F^{y}=252.6-89.2+R_{O}^{y}=0 \Rightarrow R_{O}^{y}=-163.4 \mathrm{~N} \\
& \sum M_{O}^{y}=-850 R_{E}^{z}-700(320)+300(252.6)=0 \Rightarrow R_{E}^{z}=-174.4 \mathrm{~N} \\
& \sum F^{z}=-174.4+320-252.6+R_{O}^{z}=0 \Rightarrow R_{O}^{z}=107 \mathrm{~N}
\end{aligned}
$$

$$
R_{O}=\sqrt{(-163.4)^{2}+107^{2}}=195 \mathrm{~N}
$$

$$
R_{E}=\sqrt{(-89.2)^{2}+(-174.4)^{2}}=196 \mathrm{~N}
$$

The radial loads are nearly the same at $O$ and $E$. We can use the same bearing at both locations.

$$
x_{D}=\frac{60000(1500)(60)}{10^{6}}=5400
$$

Eq. (11-6):

$$
C_{10}=1(0.196)\left\{\frac{5400}{0.02+4.439[\ln (1 / 0.9899)]^{1 / .483}}\right\}^{1 / 3}=5.7 \mathrm{kN}
$$

From Table 11-2, select an 02-12 mm deep-groove ball bearing with a basic load rating of 6.89 kN . Ans.

11-34

$$
\begin{aligned}
& R=\sqrt{0.96}=0.980 \\
& T=12\left(240 \cos 20^{\circ}\right)=2706 \mathrm{lbf} \cdot \mathrm{in} \\
& F=\frac{2706}{6 \cos 25^{\circ}}=498 \mathrm{lbf}
\end{aligned}
$$



In $x y$-plane:

$$
\Sigma M_{O}^{z}=-16(82.1)-30(210)+42 R_{C}^{y}=0
$$

$$
\begin{aligned}
& R_{C}^{y}=181 \mathrm{lbf} \\
& R_{O}^{y}=82.1+210-181=111.1 \mathrm{lbf}
\end{aligned}
$$

In $x z$-plane:

$$
\begin{aligned}
& \Sigma M_{O}^{y}=16(226)-30(451)-42 R_{C}^{z}=0 \\
& R_{C}^{z}=-236 \mathrm{lbf} \\
& R_{O}^{z}=226-451+236=11 \mathrm{lbf} \\
& R_{O}=\left(111.1^{2}+11^{2}\right)^{1 / 2}=112 \mathrm{lbf} \quad \text { Ans. } \\
& R_{C}=\left(181^{2}+236^{2}\right)^{1 / 2}=297 \mathrm{lbf} \quad \text { Ans. } \\
& x_{D}=\frac{50000(300)(60)}{10^{6}}=900 \\
& \begin{aligned}
\left(C_{10}\right)_{O} & =1.2(112)\left\{\frac{900}{0.02+4.439[\ln (1 / 0.980)]^{1 / .483}}\right\}^{1 / 3} \\
& =1860 \mathrm{lbf}=8.28 \mathrm{kN} \\
\left(C_{10}\right)_{C} & =1.2(297)\left\{\frac{900}{0.02+4.439[\ln (1 / 0.980)]^{1 / 1.483}}\right\}^{1 / 3} \\
& =4932 \mathrm{lbf}=21.9 \mathrm{kN}
\end{aligned}
\end{aligned}
$$

Bearing at $O$ : Choose a deep-groove 02-17 mm. Ans.
Bearing at $C$ : Choose a deep-groove $02-35 \mathrm{~mm}$. Ans.

11-35 Shafts subjected to thrust can be constrained by bearings, one of which supports the thrust. The shaft floats within the endplay of the second (roller) bearing. Since the thrust force here is larger than any radial load, the bearing absorbing the thrust (bearing $A$ ) is heavily loaded compared to bearing $B$. Bearing $B$ is thus likely to be oversized and may not contribute measurably to the chance of failure. If this is the case, we may be able to obtain the desired combined reliability with bearing $A$ having a reliability near 0.99 and bearing $B$ having a reliability near 1 . This would allow for bearing $A$ to have a lower capacity than if it needed to achieve a reliability of $\sqrt{0.99}$. To determine if this is the case, we will start with bearing $B$.

Bearing $B$ (straight roller bearing)

$$
\begin{aligned}
& x_{D}=\frac{30000(500)(60)}{10^{6}}=900 \\
& F_{r}=\left(36^{2}+67^{2}\right)^{1 / 2}=76.1 \mathrm{lbf}=0.339 \mathrm{kN}
\end{aligned}
$$

Try a reliability of 1 to see if it is readily obtainable with the available bearings.

Eq. (11-6): $\quad C_{10}=1.2(0.339)\left\{\frac{900}{0.02+4.439[\ln (1 / 1.0)]^{1 / 1.483}}\right\}^{3 / 10}=10.1 \mathrm{kN}$

The smallest capacity bearing from Table 11-3 has a rated capacity of 16.8 kN .
Therefore, we select the $02-25 \mathrm{~mm}$ straight cylindrical roller bearing. Ans.

## Bearing at $A$ (angular-contact ball)

With a reliability of 1 for bearing $B$, we can achieve the combined reliability goal of 0.99 if bearing $A$ has a reliability of 0.99 .

$$
\begin{aligned}
& F_{r}=\left(36^{2}+212^{2}\right)^{1 / 2}=215 \mathrm{lbf}=0.957 \mathrm{kN} \\
& F_{a}=555 \mathrm{lbf}=2.47 \mathrm{kN}
\end{aligned}
$$

Trial \#1:
Tentatively select an 02-85 mm angular-contact with $C_{10}=90.4 \mathrm{kN}$ and $C_{0}=63.0 \mathrm{kN}$.

$$
\begin{aligned}
& \frac{F_{a}}{C_{0}}=\frac{2.47}{63.0}=0.0392 \\
& x_{D}=\frac{30000(500)(60)}{10^{6}}=900
\end{aligned}
$$

Table 11-1: $\quad$ Interpolating, $X_{2}=0.56, Y_{2}=1.88$
Eq. (11-9): $\quad F_{e}=0.56(0.957)+1.88(2.47)=5.18 \mathrm{kN}$
Eq. (11-6): $\quad C_{10}=1.2(5.18)\left\{\frac{900}{0.02+4.439[\ln (1 / 0.99)]^{1 / 1.483}}\right\}^{1 / 3}$

$$
=99.54 \mathrm{kN}>90.4 \mathrm{kN}
$$

Trial \#2:
Tentatively select a 02-90 mm angular-contact ball with $C_{10}=106 \mathrm{kN}$ and $C_{0}=73.5 \mathrm{kN}$.

$$
\frac{F_{a}}{C_{0}}=\frac{2.47}{73.5}=0.0336
$$

Table 11-1: Interpolating, $X_{2}=0.56, Y_{2}=1.93$

$$
\begin{gathered}
F_{e}=0.56(0.957)+1.93(2.47)=5.30 \mathrm{kN} \\
C_{10}=1.2(5.30)\left\{\frac{900}{0.02+4.439[\ln (1 / 0.99)]^{1 / .483}}\right\}^{1 / 3}=102 \mathrm{kN}<106 \mathrm{kN} \quad \text { O.K. }
\end{gathered}
$$

Select an 02-90 mm angular-contact ball bearing. Ans.

11-36 We have some data. Let's estimate parameters $b$ and $\theta$ from it. In Fig. 11-5, we will use line $A B$. In this case, $B$ is to the right of $A$.

For $F=18 \mathrm{kN}, \quad(x)_{1}=\frac{115(2000)(60)}{10^{6}}=13.8$

This establishes point 1 on the $R=0.90$ line.


The $R=0.20$ locus is above and parallel to the $R=0.90$ locus. For the two-parameter Weibull distribution, $x_{0}=0$ and points $A$ and $B$ are related by [see Eq. (20-25)]:

$$
\begin{align*}
& x_{A}=\theta[\ln (1 / 0.90)]^{1 / b}  \tag{1}\\
& x_{B}=\theta[\ln (1 / 0.20)]^{1 / b}
\end{align*}
$$

and $x_{B} / x_{A}$ is in the same ratio as 600/115. Eliminating $\theta$,

$$
b=\frac{\ln [\ln (1 / 0.20) / \ln (1 / 0.90)]}{\ln (600 / 115)}=1.65 \quad \text { Ans } .
$$

Solving for $\theta$ in Eq. (1),

$$
\theta=\frac{x_{A}}{\left[\ln \left(1 / R_{A}\right)\right]^{1 / 1.65}}=\frac{1}{[\ln (1 / 0.90)]^{1 / 1.65}}=3.91 \quad \text { Ans. }
$$

Therefore, for the data at hand,

$$
R=\exp \left[-\left(\frac{x}{3.91}\right)^{1.65}\right]
$$

Check $R$ at point $B: x_{B}=(600 / 115)=5.217$

$$
R=\exp \left[-\left(\frac{5.217}{3.91}\right)^{1.65}\right]=0.20
$$

Note also, for point 2 on the $R=0.20$ line,

$$
\begin{aligned}
& \log (5.217)-\log (1)=\log \left(x_{m}\right)_{2}-\log (13.8) \\
& \left(x_{m}\right)_{2}=72
\end{aligned}
$$

11-37 This problem is rich in useful variations. Here is one.
Decision: Make straight roller bearings identical on a given shaft. Use a reliability goal of $(0.99)^{1 / 6}=0.9983$.

Shaft a

$$
\begin{aligned}
& F_{A}^{r}=\left(239^{2}+111^{2}\right)^{1 / 2}=264 \mathrm{lbf}=1.175 \mathrm{kN} \\
& F_{B}^{r}=\left(502^{2}+1075^{2}\right)^{1 / 2}=1186 \mathrm{lbf}=5.28 \mathrm{kN}
\end{aligned}
$$

Thus the bearing at $B$ controls.

$$
\begin{aligned}
& x_{D}=\frac{10000(1200)(60)}{10^{6}}=720 \\
& 0.02+4.439[\ln (1 / 0.9983)]^{1 / 1.483}=0.08026 \\
& C_{10}=1.2(5.28)\left(\frac{720}{0.08026}\right)^{0.3}=97.2 \mathrm{kN}
\end{aligned}
$$

Select either an 02-80 mm with $C_{10}=106 \mathrm{kN}$ or an 03-55 mm with $C_{10}=102 \mathrm{kN}$. Ans. Shaft b

$$
\begin{aligned}
& F_{C}^{r}=\left(874^{2}+2274^{2}\right)^{1 / 2}=2436 \mathrm{lbf} \text { or } 10.84 \mathrm{kN} \\
& F_{D}^{r}=\left(393^{2}+657^{2}\right)^{1 / 2}=766 \mathrm{lbf} \quad \text { or } \quad 3.41 \mathrm{kN}
\end{aligned}
$$

The bearing at $C$ controls.

$$
\begin{aligned}
& x_{D}=\frac{10000(240)(60)}{10^{6}}=144 \\
& C_{10}=1.2(10.84)\left(\frac{144}{0.08026}\right)^{0.3}=123 \mathrm{kN}
\end{aligned}
$$

Select either an 02-90 mm with $C_{10}=142 \mathrm{kN}$ or an 03-60 mm with $C_{10}=123 \mathrm{kN}$. Ans.
Shaft c

$$
\begin{aligned}
& F_{E}^{r}=\left(1113^{2}+2385^{2}\right)^{1 / 2}=2632 \mathrm{lbf} \quad \text { or } \quad 11.71 \mathrm{kN} \\
& F_{F}^{r}=\left(417^{2}+895^{2}\right)^{1 / 2}=987 \mathrm{lbf} \quad \text { or } \quad 4.39 \mathrm{kN}
\end{aligned}
$$

The bearing at $E$ controls.

$$
\begin{aligned}
& x_{D}=\frac{10000(80)(60)}{10^{6}}=48 \\
& C_{10}=1.2(11.71)\left(\frac{48}{0.08026}\right)^{0.3}=95.7 \mathrm{kN}
\end{aligned}
$$

Select an 02-80 mm with $C_{10}=106 \mathrm{kN}$ or an $03-60 \mathrm{~mm}$ with $C_{10}=123 \mathrm{kN}$. Ans.

11-38 Express Eq. (11-1) as

$$
F_{1}^{a} L_{1}=C_{10}^{a} L_{10}=K
$$

For a ball bearing, $a=3$ and for an 02-30 mm angular contact bearing, $C_{10}=20.3 \mathrm{kN}$.

$$
K=(20.3)^{3}\left(10^{6}\right)=8.365\left(10^{9}\right)
$$

At a load of 18 kN , life $L_{1}$ is given by:

$$
L_{1}=\frac{K}{F_{1}^{a}}=\frac{8.365\left(10^{9}\right)}{18^{3}}=1.434\left(10^{6}\right) \mathrm{rev}
$$

For a load of 30 kN , life $L_{2}$ is:

$$
L_{2}=\frac{8.365\left(10^{9}\right)}{30^{3}}=0.310\left(10^{6}\right) \mathrm{rev}
$$

In this case, Eq. (6-57) - the Palmgren-Miner cycle-ratio summation rule - can be expressed as

$$
\frac{l_{1}}{L_{1}}+\frac{l_{2}}{L_{2}}=1
$$

Substituting,

$$
\begin{aligned}
& \frac{200000}{1.434\left(10^{6}\right)}+\frac{l_{2}}{0.310\left(10^{6}\right)}=1 \\
& l_{2}=0.267\left(10^{6}\right) \mathrm{rev} \quad \text { Ans. }
\end{aligned}
$$

## 11-39 Total life in revolutions

Let:
$l=$ total turns
$f_{1}=$ fraction of turns at $F_{1}$
$f_{2}=$ fraction of turns at $F_{2}$
From the solution of Prob. 11-38, $L_{1}=1.434\left(10^{6}\right)$ rev and $L_{2}=0.310\left(10^{6}\right)$ rev.
Palmgren-Miner rule:

$$
\frac{l_{1}}{L_{1}}+\frac{l_{2}}{L_{2}}=\frac{f_{1} l}{L_{1}}+\frac{f_{2} l}{L_{2}}=1
$$

from which

$$
\begin{aligned}
l & =\frac{1}{f_{1} / L_{1}+f_{2} / L_{2}} \\
l & =\frac{1}{\left\{0.40 /\left[1.434\left(10^{6}\right)\right]\right\}+\left\{0.60 /\left[0.310\left(10^{6}\right)\right]\right\}} \\
& =451585 \mathrm{rev} \text { Ans. }
\end{aligned}
$$

Total life in loading cycles

$$
\begin{aligned}
& 4 \mathrm{~min} \text { at } 2000 \mathrm{rev} / \mathrm{min}=8000 \mathrm{rev} / \mathrm{cycle} \\
& 6 \mathrm{~min} \text { at } 2000 \mathrm{rev} / \mathrm{min}=12000 \mathrm{rev} / \mathrm{cycle}
\end{aligned}
$$

Total rev/cycle $=8000+12000=20000$

$$
\frac{451585 \mathrm{rev}}{20000 \mathrm{rev} / \mathrm{cycle}}=22.58 \text { cycles } \text { Ans. }
$$

## Total life in hours

$$
\left(10 \frac{\mathrm{~min}}{\text { cycle }}\right)\left(\frac{22.58 \text { cycles }}{60 \mathrm{~min} / \mathrm{h}}\right)=3.76 \mathrm{~h} \quad \text { Ans. }
$$

11-40

$$
\begin{aligned}
& F_{r A}=560 \mathrm{lbf} \\
& F_{r B}=1095 \mathrm{lbf} \\
& F_{a e}=200 \mathrm{lbf} \\
& x_{D}=\frac{L_{D}}{L_{R}}=\frac{40000(400)(60)}{90\left(10^{6}\right)}=10.67 \\
& R=\sqrt{0.90}=0.949
\end{aligned}
$$

Eq. (11-15): $\quad F_{i A}=\frac{0.47 F_{r A}}{K_{A}}=\frac{0.47(560)}{1.5}=175.5 \mathrm{lbf}$
Eq. (11-15): $\quad F_{i B}=\frac{0.47 F_{r B}}{K_{B}}=\frac{0.47(1095)}{1.5}=343.1 \mathrm{lbf}$

$$
F_{i A} \leq ? \geq\left(F_{i B}+F_{a e}\right)
$$

$175.5 \mathrm{lbf} \leq(343.1+200)=543.1 \mathrm{lbf}$, so Eq. (11-16) applies.
We will size bearing $B$ first since its induced load will affect bearing $A$, but is not itself affected by the induced load from bearing $A$ [see Eq. (11-16)].

From Eq. (11-16b), $F_{e B}=F_{r B}=1095 \mathrm{lbf}$.
Eq. (11-7): $\quad F_{R B}=1.4(1095)\left(\frac{10.67}{4.48(1-0.949)^{1 / 1.5}}\right)^{3 / 10}=3607 \mathrm{lbf} \quad$ Ans.
Select cone 32305, cup 32305, with 0.9843 in bore, and rated at 3910 lbf with $K=1.95$. Ans.

With bearing $B$ selected, we re-evaluate the induced load from bearing $B$ using the actual value for $K$.
Eq. (11-15): $\quad F_{i B}=\frac{0.47 F_{r B}}{K_{B}}=\frac{0.47(1095)}{1.95}=263.9 \mathrm{lbf}$
Find the equivalent radial load for bearing $A$ from Eq. (11-16), which still applies.
Eq. $(11-16 a): \quad F_{e A}=0.4 F_{r A}+K_{A}\left(F_{i B}+F_{a e}\right)$

$$
F_{e A}=0.4(560)+1.5(263.9+200)=920 \mathrm{lbf}
$$

$F_{e A}>F_{r A}$
Eq. (11-7): $\quad F_{R A}=1.4(920)\left(\frac{10.67}{4.48(1-0.949)^{1 / 1.5}}\right)^{3 / 10}=3030 \mathrm{lbf}$
Tentatively select cone M86643, cup M86610, with 1 in bore, and rated at 3250 lbf with $K=1.07$. Iterating with the new value for $K$, we get $F_{e A}=702 \mathrm{lbf}$ and $F_{r A}=2312 \mathrm{lbf}$. Ans.

By using a bearing with a lower $K$, the rated load decreased significantly, providing a higher than requested reliability. Further examination with different combinations of bearing choices could yield additional acceptable solutions.

11-41 The thrust load on shaft $C D$ is from the axial component of the force transmitted through the bevel gear, and is directed toward bearing $C$. By observation of Fig. 11-14, direct mounted bearings would allow bearing $C$ to carry the thrust load. Ans.

From the solution to Prob. 3-74, the axial thrust load is $F_{a e}=362.8 \mathrm{lbf}$, and the bearing radial forces are $F_{C x}=287.2 \mathrm{lbf}, F_{C z}=500.9 \mathrm{lbf}, F_{D x}=194.4 \mathrm{lbf}$, and $F_{D z}=307.1 \mathrm{lbf}$. Thus, the radial forces are

$$
\begin{aligned}
& F_{r C}=\sqrt{287.2^{2}+500.9^{2}}=577 \mathrm{lbf} \\
& F_{r D}=\sqrt{194.4^{2}+307.1^{2}}=363 \mathrm{lbf}
\end{aligned}
$$

The induced loads are
Eq. (11-15): $\quad F_{i C}=\frac{0.47 F_{r C}}{K_{C}}=\frac{0.47(577)}{1.5}=181 \mathrm{lbf}$
Eq. (11-15): $\quad F_{i D}=\frac{0.47 F_{r D}}{K_{D}}=\frac{0.47(363)}{1.5}=114 \mathrm{lbf}$
Check the condition on whether to apply Eq. (11-16) or Eq. (11-17), where bearings $C$ and $D$ are substituted, respectively, for labels $A$ and $B$ in the equations.

$$
\begin{aligned}
& F_{i C} \leq ? \geq F_{i D}+F_{a e} \\
& 181 \mathrm{lbf}<114+362.8=476.8 \mathrm{lbf} \text {, so Eq. }(11-16) \text { applies }
\end{aligned}
$$

Eq. $(11-16 a): \quad F_{e C}=0.4 F_{r C}+K_{C}\left(F_{i D}+F_{a e}\right)$

$$
=0.4(577)+1.5(114+362.8)=946 \mathrm{lbf}>F_{r C,} \text {, so use } F_{e C}
$$

Assume for tapered roller bearings that the specifications for Manufacturer 1 on p. 608 are applicable.

$$
\begin{aligned}
& x_{D}=\frac{L_{D}}{L_{R}}=\frac{10^{8}}{90\left(10^{6}\right)}=1.11 \\
& R=\sqrt{0.90}=0.949
\end{aligned}
$$

Eq. (11-7): $\quad F_{R C}=1(946)\left(\frac{1.11}{4.48(1-0.949)^{1 / 1.5}}\right)^{3 / 10}=1130 \mathrm{lbf} \quad$ Ans.
Eq. (11-16b): $\quad F_{e D}=F_{r D}=363 \mathrm{lbf}$
Eq. (11-7): $\quad F_{R D}=1(363)\left(\frac{1.11}{4.48(1-0.949)^{1 / 1.5}}\right)^{3 / 10}=433 \mathrm{lbf} \quad$ Ans.

11-42 The thrust load on shaft $A B$ is from the axial component of the force transmitted through the bevel gear, and is directed to the right. By observation of Fig. 11-14, indirect mounted bearings would allow bearing $A$ to carry the thrust load. Ans.

From the solution to Prob. 3-76, the axial thrust load is $F_{a e}=92.8 \mathrm{lbf}$, and the bearing radial forces are $F_{A y}=639.4 \mathrm{lbf}, F_{A z}=1513.7 \mathrm{lbf}, F_{B y}=276.6 \mathrm{lbf}$, and $F_{B z}=705.7 \mathrm{lbf}$. Thus, the radial forces are

$$
\begin{aligned}
& F_{r A}=\sqrt{639.4^{2}+1513.7^{2}}=1643 \mathrm{lbf} \\
& F_{r B}=\sqrt{276.6^{2}+705.7^{2}}=758 \mathrm{lbf}
\end{aligned}
$$

The induced loads are
Eq. (11-15): $\quad F_{i A}=\frac{0.47 F_{r A}}{K_{A}}=\frac{0.47(1643)}{1.5}=515 \mathrm{lbf}$
Eq. (11-15): $\quad F_{i B}=\frac{0.47 F_{r B}}{K_{B}}=\frac{0.47(758)}{1.5}=238 \mathrm{lbf}$
Check the condition on whether to apply Eq. (11-16) or Eq. (11-17).

$$
\begin{aligned}
& F_{i A} \leq ? \geq F_{i B}+F_{a e} \\
& 515 \mathrm{lbf}>238+92.8=330.8 \mathrm{lbf} \text {, so Eq.(11-17) applies }
\end{aligned}
$$

Notice that the induced load from bearing $A$ is sufficiently large to cause a net axial force to the left, which must be supported by bearing $B$.

Eq. $(11-17 a): \quad F_{e B}=0.4 F_{r B}+K_{B}\left(F_{i A}-F_{a e}\right)$

$$
=0.4(758)+1.5(515-92.8)=937 \mathrm{lbf}>F_{r B,} \text {, so use } F_{e B}
$$

Assume for tapered roller bearings that the specifications for Manufacturer 1 on p. 608 are applicable.

$$
\begin{aligned}
& x_{D}=\frac{L_{D}}{L_{R}}=\frac{500\left(10^{6}\right)}{90\left(10^{6}\right)}=5.56 \\
& R=\sqrt{0.90}=0.949
\end{aligned}
$$

Eq. (11-7): $\quad F_{R B}=1(937)\left(\frac{5.56}{4.48(1-0.949)^{1 / 1.5}}\right)^{3 / 10}=1810 \mathrm{lbf} \quad$ Ans.
Eq. (11-16b): $\quad F_{e A}=F_{r A}=1643 \mathrm{lbf}$
Eq. (11-7): $\quad F_{R A}=1(1643)\left(\frac{5.56}{4.48(1-0.949)^{1 / 1.5}}\right)^{3 / 10}=3180 \mathrm{lbf} \quad$ Ans.

11-43 The lower bearing is compressed by the axial load, so it is designated as bearing $A$.

$$
\begin{aligned}
& F_{r A}=25 \mathrm{kN} \\
& F_{r B}=12 \mathrm{kN} \\
& F_{a e}=5 \mathrm{kN}
\end{aligned}
$$

Eq. (11-15): $\quad F_{i A}=\frac{0.47 F_{r A}}{K_{A}}=\frac{0.47(25)}{1.5}=7.83 \mathrm{kN}$
Eq. (11-15): $\quad F_{i B}=\frac{0.47 F_{r B}}{K_{B}}=\frac{0.47(12)}{1.5}=3.76 \mathrm{kN}$
Check the condition on whether to apply Eq. (11-16) or Eq. (11-17)

$$
F_{i A} \leq ? \geq F_{i B}+F_{a e}
$$

$7.83 \mathrm{kN}<3.76+5=8.76 \mathrm{kN}$, so Eq.(11-16) applies

Eq. $(11-16 a): \quad F_{e A}=0.4 F_{r A}+K_{A}\left(F_{i B}+F_{a e}\right)$

$$
\begin{aligned}
& =0.4(25)+1.5(3.76+5)=23.1 \mathrm{kN}<F_{r A}, \text { so use } F_{r A} \\
L_{D} & =(250 \mathrm{rev} / \mathrm{min})\left(\frac{60 \mathrm{~min}}{\mathrm{hr}}\right)\left(\frac{8 \mathrm{hr}}{\text { day }}\right)\left(\frac{5 \text { day }}{\text { week }}\right)\left(\frac{52 \text { weeks }}{\mathrm{yr}}\right)(5 \mathrm{yrs}) \\
& =156\left(10^{6}\right) \mathrm{rev}
\end{aligned}
$$

Assume for tapered roller bearings that the specifications for Manufacturer 1 on p. 608 are applicable.
Eq. (11-3): $\quad F_{R A}=a_{f} F_{D}\left[\frac{L_{D}}{L_{R}}\right]^{3 / 10}=1.2(25)\left[\frac{156\left(10^{6}\right)}{90\left(10^{6}\right)}\right]^{3 / 10}=35.4 \mathrm{kN} \quad$ Ans.
Eq. $(11-16 b): \quad F_{e B}=F_{r B}=12 \mathrm{kN}$

Eq. (11-3): $\quad F_{R B}=1.2(12)\left[\frac{156}{90}\right]^{3 / 10}=17.0 \mathrm{kN} \quad$ Ans.

11-44 The left bearing is compressed by the axial load, so it is properly designated as bearing $A$.

$$
\begin{aligned}
& F_{r A}=875 \mathrm{lbf} \\
& F_{r B}=625 \mathrm{lbf} \\
& F_{a e}=250 \mathrm{lbf}
\end{aligned}
$$

Assume $K=1.5$ for each bearing for the first iteration. Obtain the induced loads.
Eq. (11-15): $\quad F_{i A}=\frac{0.47 F_{r A}}{K_{A}}=\frac{0.47(875)}{1.5}=274 \mathrm{lbf}$
Eq. (11-15): $\quad F_{i B}=\frac{0.47 F_{r B}}{K_{B}}=\frac{0.47(625)}{1.5}=196 \mathrm{lbf}$
Check the condition on whether to apply Eq. (11-16) or Eq. (11-17).

$$
\begin{aligned}
& F_{i A} \leq ? \geq F_{i B}+F_{a e} \\
& 274 \mathrm{lbf}<196+250 \mathrm{lbf}, \text { so Eq.(11-16) applies }
\end{aligned}
$$

We will size bearing $B$ first since its induced load will affect bearing $A$, but it is not affected by the induced load from bearing $A$ [see Eq. (11-16)].

From Eq. (11-16b), $F_{e B}=F_{r B}=625 \mathrm{lbf}$.
Eq. (11-3): $\quad F_{R B}=a_{f} F_{D}\left[\frac{L_{D}}{L_{R}}\right]^{3 / 10}=1(625)\left[\frac{90000(150)(60)}{90\left(10^{6}\right)}\right]^{3 / 10}$

$$
F_{R B}=1208 \mathrm{lbf}
$$

Select cone 07100 , cup 07196 , with 1 in bore, and rated at 1570 lbf with $K=1.45$. Ans.
With bearing $B$ selected, we re-evaluate the induced load from bearing $B$ using the actual value for $K$.
Eq. (11-15): $\quad F_{i B}=\frac{0.47 F_{r B}}{K_{B}}=\frac{0.47(625)}{1.45}=203 \mathrm{lbf}$
Find the equivalent radial load for bearing $A$ from Eq. (11-16), which still applies.

Eq. $(11-16 a): \quad F_{e A}=0.4 F_{r A}+K_{A}\left(F_{i B}+F_{a e}\right)$

$$
\begin{aligned}
& \quad=0.4(875)+1.5(203+250)=1030 \mathrm{lbf} \\
& F_{e A}>F_{r A}
\end{aligned}
$$

Eq. (11-3): $\quad F_{R A}=a_{f} F_{D}\left[\frac{L_{D}}{L_{R}}\right]^{3 / 10}=1(1030)\left[\frac{90000(150)(60)}{90\left(10^{6}\right)}\right]^{3 / 10}$

$$
F_{R A}=1990 \mathrm{lbf}
$$

Any of the bearings with 1-1/8 in bore are more than adequate. Select cone 15590, cup 15520 , rated at 2480 lbf with $K=1.69$. Iterating with the new value for $K$, we get $F_{e A}=$ 1120 lbf and $F_{r A}=2160 \mathrm{lbf}$. The selected bearing is still adequate. Ans.

